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Copula-Based Modelling of Drought Severity-Duration-Frequency Relationship of Sokoto-Rima-River Basin, Nigeria

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Abstract

Effective drought management in vulnerable semi-arid regions necessitates a comprehensive understanding of drought characteristics and their interdependencies, crucial for proactive water resource planning and risk assessment. This study addresses the critical gap in bivariate drought modelling for the Sokoto-Rima River Basin, Northern Nigeria, by employing a copula-based approach to analyse the Severity-Duration-Frequency (SDF) relationship. Utilising historical monthly rainfall data (1945-2015), the Standardised Precipitation Index (SPI-6) was identified as the optimal timescale for hydrological drought characterisation. Drought severity and duration marginal distributions were best fitted by the Generalised Extreme Value (GEV) and Lognormal distributions, respectively. A strong positive correlation (Kendall's τ=0.7599) was established between drought severity and duration. The Survival (BB8) copula emerged as the most efficient dependence structure, demonstrating superior goodness-of-fit (lowest RMSE: 0.0023, AIC: -82.9876, BIC: -79.0850) and accurately capturing the joint probability distribution. Tail dependence analysis using the Capéraa, Fougères, and Genest (CFG) estimator revealed a weak upper tail dependence coefficient (0.001), indicating that while individual extreme events are common, the simultaneous occurrence of extremely severe and prolonged droughts is historically less probable. The derived bivariate SDF curves offer a robust tool for probabilistic risk assessment, quantifying the likelihood of specific drought severity-duration combinations for various return periods. This research provides invaluable insights for developing advanced drought early warning systems and informing sustainable water resource management and climate change adaptation strategies in data-scarce, drought-prone environments.

Keyword: *Drought, Climate, Copula, Nigeria, Severity, Duration.*

1.0 Introduction

Climate models indicate that increased global temperatures and changing precipitation trends will affect the frequency and strength of extreme weather phenomena [1]. Accordingly, drought evaluation has gained a broad range of attention as a serious global issue by water resource experts. Droughts and floods, in this regard, are considered major environmental disasters that impede the implementation of sustainable water resource management and development [2]. Droughts are usually defined as a prolonged period of substantially reduced precipitation over a given area leading to perennial water scarcity across the atmospheric, surface, and groundwater regimes. It is characterised by its complexity, especially it changing states attribute that is from meteorological to hydrological and ultimately to agricultural drought [3]. In contrast to other rapid-onset natural disasters, drought gradually manifests as extended periods of precipitation deficits. Such precedent climatic deviations would result in catastrophic environmental, agricultural, and introduce socioeconomic effects [3]. Historically, northern Nigeria had a high incidence of extended droughts, which often lead to havoc in agriculture and food security. Instances of drought were recorded in 1903 and 1911 to 1914 [4]. Similar drought event was observed in 1951 to 1954, 1972 to 1973; and 1984 to 1985 Eze [5], and reiterated in 2007 and 2011 [6]. The impacts of these droughts worsened greatly as a result of rapid population growth, intensified agricultural activities, overgrazing and acute poverty [7]. The insufficient rainfall periods stress the agro-based economy hence supported poor harvests, crop failures and food shortage, consequent on severe losses in livestock [5].

Drought events are naturally unpredictable, being subject to variability in rainfall and discharge. Hence, the effective analysis of droughts necessitates the application of probability theory and stochastic modelling techniques. Effective water resource management is especially critical in this context, as such an analysis gives a clearer picture of drought conditions and water shortages affected by climatic shifts [8]. Chowdhury and Singh [9] report an increasing interest in undertaking research and developing models that accurately reflect the complexities of drought occurrences. Considerable the advances made over the last couple of decades in hydrological modelling due to the increased capability of computational technology [9]. Newer approaches employing statistical inference and pattern recognition are being explored to gain knowledge of both space and time change of hydrological processes and their surmised interrelationships amidst various hydrological factors. Key drought features include

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severity, duration, and frequency, are mostly used to develop modern hydrological design and management frameworks [9].

Chebana and Ouarda [10] have suggested that the importance of multivariate analysis in hydrological modelling and risk assessment is increasingly being recognised. By implication, multivariate analysis considers a greater number of variable influences, as opposed to traditional univariate approaches, allowing their interactions to emerge with consideration given towards the hydrological processes involved and their implications for design and management considerations [11]. In view of the interdependence of drought characteristics like severity, duration, and frequency, it becomes imperative to have a reasonably exhaustive modelling framework that captures these dependencies [12]. The inter-relationships that characterise the three have to be taken into account for a proper description of droughts by means of a joint probability distribution of these variables [12]. In this regard, copula model provides an excellent statistical methodology to this end, because it captures the relationship between multiple drought variables by linking their respective marginal distributions, no matter the type of distribution the variables may be drawn from [13]. This is especially relevant with regard to the nonlinear character of hydrological processes of drought. Hence, modelling using copula techniques has gained wide acceptance and application in predicting both drought and flood conditions [14] Copula functions have been used to study the interactions of drought quantities. Among them is a study at Sharafkhaneh Station in Iran, where several copula models were used to specify the dependency structure of drought characteristics. According to the study findings, the Galambos copula appeared to be most effective for jointly capturing of duration and severity [15]. Another study applied Clayton copula in analysing drought length and intensity and it was found that drought events in Iran, are the moderate class [16]. Kao *et al.* [17] in the Midwestern United States adopted copula-based approaches to evaluate the spatial and temporal dynamics of drought; this study also introduced an alternative approach to the Palmer Drought Severity Index, with Joint Deficit Index (JDI), which showed that the index was capable of providing a more monthly accurate evaluation compared to that of drought evolution and improving the analysis of recovery processes.

The tail dependence test is undoubtedly the crux of extreme event modelling, but it lacks significant coverage in most of the literature in drought studies. Although tail dependence is crucial for engineering risk assessment for plausible drought analysis, it greatly assists in identifying the best copula in drought modelling. In the phase of this surmised importance, there is perceived cooled or sluggishness towards fully integrated it concept in drought quantification. In the general horizon of drought analysis in Northern Nigeria, there is complete dearth of application of copula bivariate modelling. In the light of these; there is observed absent of extensive study that systematically meld tail dependence concept in describing drought scenario of Sokoto-Rima River Basin nor any trace of documented evidence that highlighted the implication of copula-based Severity-Duration-Frequency (S-D-F) curve in the region. In recognition, this research bridged the missing link considering the fact S-D-F curve provides very useful insights for engineering drought risk assessments and also forms a major tool for water resource planning and management. In the overall, the aim of this study is to modelling drought S-D-F relationship of Sokoto-rima-river basin using bivariate Copula model.

2.0 Material and Methods

2.1 Study Location and Hydrometeorology

The studied location covered basically Hydrological Area One (HA1). Sokoto-rima-river basin generally lies between latitude 10°N and 14°N and longitude 4°E and 9°5'E. It is located with the semi-arid belt of Nigeria under the Sudan savannah as shown in Figure 1. Average annual rainfall varies from 364 mm to 970 mm. Four major dams Goronyo, Bakolori, Zobe, and Jibia cater most of the water for irrigation, domestic, and other uses in the basin. Rainfall data were collected from gauging stations distributed across the studied basin which includes Sokoto, Goronyo, Gusau, Bakolori, Katsina, Zobe, and Jibia. The data spans from 1945 to 2015, representing 45 years, for the evaluation of long-term hydrometeorological phenomenon. In addition, the monthly rainfall records were sourced from the Nigerian Meteorological Agency (NiMet) and the Sokoto-Rima River Basin Development Authority (SRRBDA) across four of the states.

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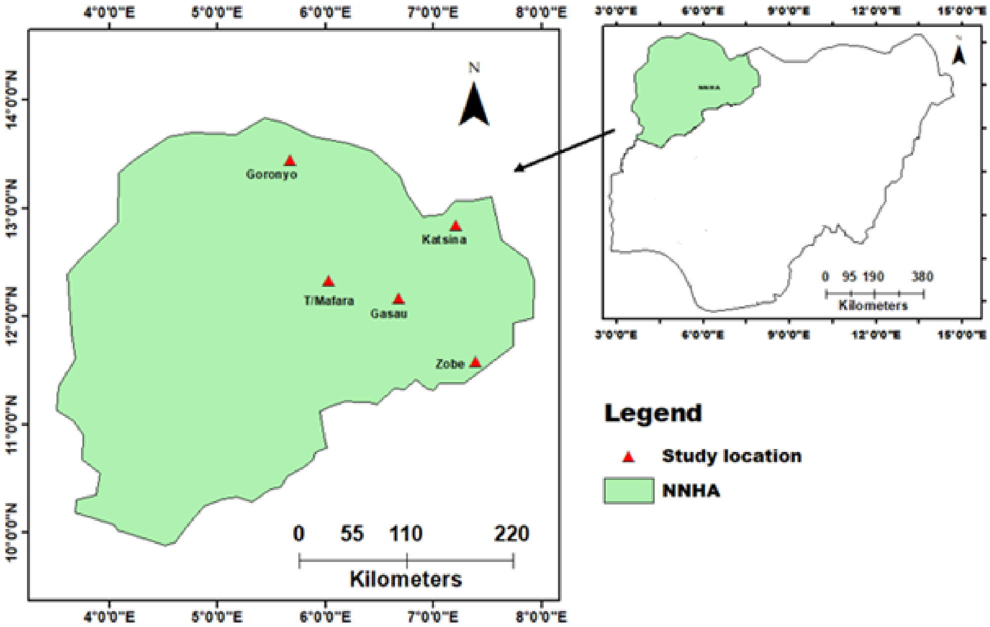

Figure 1: Map of the study area

2.2 Drought Series Computation

The SPI, originally developed by McKee reported by Otache [18], was used in this study to quantify and characterise drought conditions. The SPI was selected because of its simplicity with regard to data necessities and several key advantages such as statistical reliability and the usage of protocol to study short- and long-term hydro-climatic impacts across multiple time scales. The SPI being probabilistic in nature means that one can easily analyse drought based on various variables and locations, which provides interpretations based on return periods or recurrence intervals; hence, it is popular as a standard approach to drought evaluation. Since severity is understood to be the parameter most sensitive to spatial variations in rainfall patterns or anomalies, it becomes the subject of analysis for this research as shown in equation 1

$$S = -\sum_{i=1}^{D} SPI_i \tag{1}$$

Where D is duration, and S is severity.

2.3 Selection of Appropriate Probability Distribution Models

An extensive analysis was undertaken to identify which probability distribution would best represent drought severity and duration as computed by SPI. Thus, four probability distributions (Extreme Value, Lognormal, Gamma, and Weibull) were assessed by means of different statistical tests to check which distributions were epitomic. The evaluations being essentially objective were carried out by using various criteria of goodness of fit for applying parametric estimated distributions and empirical data. Statistical tests for evaluation of this nature included the Cramer-von Mises test, Kolmogorov-Smirnov test, and Anderson-Darling test depending on their strength for measuring each of the best-fit distributions against the real-world observations. In addition, model selection criteria were on the basis test of statistics. For the validation of the work done on fitting the distributions in accordance with Chukwu [19].

2.4 Theory of Copula Function

In this context, in line with Chukwu [24] u and v refer to univariate cumulative distribution functions of duration and severity series. Typical of Archimedean family being the common copula used in drought analysis, equation 2a, 2b and 2c provides computational framework for the bivariate copula models.

$$c(u,v) = \varphi^{-1}[\varphi(u) + \cdots + \varphi(v)] \tag{2a}$$

Here, $\varphi(t)$ represents the copula generator and φ^{-1} refers to its inverse. Moreover, a bivariate copula is identified as an Extreme Value (EV) copula if there exists a convex function called the Pickands dependence function, A: [0, 1] → [1/2, 1], satisfying a few conditions. These conditions include A(0) = A(1)=1 and for any $t \in [0,1]$, max $\{t, 1-t\} \leq A(t) \leq 1$ which is given in Nelsen [20].

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$$c(u,v) = exp\left[\log(uv)\, A(\frac{\log v}{\log(vu)})\right] \quad (2b)$$

for all $(u,v) \in t^2$. In specific, assuming At=1 at that point u, are independent; also, if At=max$t$, 1$-t$, then (u,v) are seamlessly dependent (or co-monotonic). Contrariwise, given a bivariate EV copula C, the corresponding Pickands dependence equation (A) is given by the following:

$$A(t) = -InC\left(e^{-(1-t),e^{t}}\right) \quad (2c)$$

Where $t \in \{0,1\}$
For the model selection process to remain robust, goodness-of-fit statistics was employed to ascertain the fitness of the choose model.

2.4.1 Selection of Appropriate Copula Model

Determining the degree of relationship between variables of paramount for modelling multivariate extreme values. As traditional measures, the advantages and disadvantages of Pearson's correlation coefficient (rho) are manifested when used for bivariate distributions. An important limitation of Pearson's ϱ is its insistence on a linear connection. Thus, it fails in applications where more-complex dependencies exist. Furthermore, applicability when faced with heavy-tailed distributions is undermined, so it has little relevance to extreme value theory. Therefore, there are suitable nonparametric techniques, such as Kendall's tau and Spearman's rank correlation (rho). Among these, Kendall's tau is the most favoured for copula-based modelling because it offers credible information on the dependence of two random variables. In contrast to Pearson's correlation, which assumes a linear relationship, Kendall's tau assesses association based on the ranks of the data points. In fact, Kendall's tau is defined in the theoretical sense as half the probability of randomly selecting two data pairs with the same timestamp, one of which is concordant and the other discordant. The mathematical expression for that is given below in equation 3a,

$$\tau = P[(X_1 - X_2)(Y_1 - Y_2) > 0\,] - P[(X_1 - X_2)(Y_1 - Y_2) < 0\,] \quad (3a)$$

$P[(X_1 - X_2)(Y_1 - Y_2) > 0\,]\ and\ P[(X_1 - X_2)(Y_1 - Y_2) < 0$ represent the probability of concordance and discordance which refer to the probability of events on the distribution of pairs. The pair (x(i) y(i)) will betermed concordant whenever (x-i-x-j) (y-i-y-j)>0 and discordant whenever (x-i-x-j) (y-i-y-j)<0. Given a random sample of size n of bivariate observations (x-1, y-1),(x-2, y-2),…(x-n,y-n), sample-based estimation of Kendall's τ will be based on the following equations,

$$\tau = \binom{n}{2}^{-1} \sum_{1\le i<j\le n} sgn\left[(x_i - x_j)(y_i - y_j)\right] \quad (3b)$$

Where $sgn\ i, j$ = 1, 2,…,n, and

$$(\varphi) = \begin{cases} 1\ if \quad \varphi \ge 0 \\ 2\ \ if \quad \varphi = 0 \\ -1\ if\ \varphi < \ 0 \end{cases} \quad (3)$$

The selection of appropriate copula functions was dependent mainly on its individual ability to cope with, that is to evaluate various degrees of dependence, quantifiable with Kendall's Tau. Gumbel-Hougaard copula has strong positive associations. There are other copulas, Ali-Mikhail-Haq and Farlie-Gumbel-Morgenstern, that have constraints, with ranges −0.1807<τ<0.3333-0.1807 <and −2/9≤τ≤2/9, respectively, in their consideration of dependence. Conversely, both the Clayton and Frank copulas can be used to capture a combination of positive and negative dependency structure [19]. Theta, the dependence parameter, essentially measures the strength of the relationship between the two variables u and v. The functions of these copulas are summarised in Table 1 according to their limits of dependence parameters.

Table 1: Selected Copula and the range of their dependence parameters

Family	$c(u,v)$	Range of θ
Ali-Mikhail-Haq	$\frac{uv}{1-\theta(1-u)(1-u)}$	$-1 \le \theta \le 1$
Clayton	$(u^{-\theta} + v^{\theta} - 1)^{-1/\theta}$	$\theta \ge 0$

Farlie-Gumbel-Morgenstern	$uv[1+\theta(1-u)(1-v)]$	$-1 \le \theta \le 1$
Frank	$-\frac{1}{\theta} In\left[1+\frac{\left(e^{-\theta u}\right)\left\|e^{-\theta v}-1\right\|}{e^{-\theta-1}}\right]$	$\theta \neq 0$
Galambos	$uvexp\left\{\left[\left(-Inu^{-\theta}\right)+\left(-Inv^{\theta}\right)\right]^{-\frac{1}{\theta}}\right\}$	$\theta \ge 0$
Gumbel-Hougaard	$exp\left\{-\left[\left(-Inu^{-\theta}\right)+\left(-Inv^{\theta}\right)\right]^{\frac{1}{\theta}}\right\}$	$\theta \ge 1$
Survival	$u+v-1+C(1-u,1-v)$	$\theta \ge 0$

Sources: Chukwu **[**190]

In frequency analysis of extreme hydrological occurrences such as droughts, tail dependence quantifies the degree of association between extreme values of two dependent variables, especially in the upper or lower tails of their distribution. Ignoring tail dependence of multiple hydrological variables may lead to significant uncertainties in estimating extreme quantiles, thereby conceiving erroneous designs. Earlier studies show that tail dependency must be included in bivariate frequency analyses, especially while assessing risks through copula-based models. The studies show that predictive skill in hydrological forecasting benefits from incorporating tail dependence. Simulations across copula families have allowed researchers to compare different nonparametric estimators of tail dependence coefficients. Understanding tail dependence is important for fitting the chosen copula model to the interdependence structure of extreme hydrological processes [20]. In bivariate analysis, the upper and lower tail dependence coefficients, λ_u and λ_L , represent the existing dependency structure at extremes and is defined mathematically by:

$$\lambda_u = \lim_{n \to 1^-} P\{F_X(x) > t | F_Y(y) > t\} \tag{3d}$$

$$\lambda_L = \lim_{n \to 0^-} P\{F_X(x) < t | F_Y(y) < t\} \tag{3e}$$

F_X and F_Y denote the cumulative distribution functions corresponding to the random variables X and Y, respectively, while t represents a fixed value of a standard uniform variable. A bivariate distribution function is said to exhibit upper or lower tail dependence when $0 < \lambda_u \le 1 (0 < \lambda_L \le 1)$. On the other hand, the distribution is considered upper tail independent if $\lambda_u = 0$ or lower tail independent when $(\lambda_L = 0)$.

$$\lambda_u = \lim_{n \to 1^-} \frac{1-2t+C(t,t)}{1-t} \tag{3f}$$

$$\lambda_u = \lim_{n \to 0^+} \frac{C(t,t)}{t} \tag{3g}$$

In essence, the choice of copula is linked with tail properties, rather than the choice of marginal distributions. Some copulas are said to show dependence towards the extremes, while others do not. For example, the Gaussian copula shows no dependence on either upper or lower tails. The non-Gaussian copulas, however, exhibit differing tail dependence characteristics. The Clayton copula has lower tail dependence but no upper tail dependence, while Ali-Mikhail-Haq, Farlie-Gumbel-Morgenstern, Frank, and Plackett copulas do not demonstrate dependence on either tail [20]. Contrasting with the rest, Galambos and Gumbel-Hougaard copulas show evidence of dependence on upper tails but not on lower ones. The present research emphasises that upper tail dependence is essential for evaluating the concurrent occurrence of extreme hydrological events. In contrast to other measures like Kendall's tau, estimating upper tail dependence creates additional challenges. For this reason, three influential nonparametric estimators are given: logarithmic (LOG) estimator, secant (SEC) estimator, and the Capéraa, Fougères, and Genest (CFG) estimator. In this study, the CFG estimator is chosen above other LOG and SEC estimators, as it quantifies upper tail dependence without the use of a defined threshold. The upper tail dependence coefficient calculated by the CFG estimator follows a mathematical formulation to express the degree of dependence of extreme events [21].

$$\widehat{\lambda_u^{CFG}} = 2 - 2exp\left\{\frac{1}{n}\sum_{i=1}^{n} log\left[\frac{\sqrt{log\frac{1}{u_1}log\frac{1}{v_1}}}{log\frac{1}{max(u_1v_1)}}\right]\right\} \tag{3h}$$

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The variables u-1 and v-1 denote the cumulative distribution functions of the interest variables, such that u-1=F-x (x-1) and v-1=F-Y (y-1). Even if extreme value copulas are usually being regarded as appropriate proxies in modelling dependence, work done by Frahm et al. **[**22**]** suggests that this form of a copula is not very rigid. Their results state that it is even possible to derive the estimator when a copula does not belong to the extreme value family strictly [22]. This study examined several copula models such as Clayton, Ali-Mikhail-Haq, Farlie-Gumbel-Morgenstern, Frank, Galambos, Gumbel-Hougaard, and Survival copulas; these models are characterised by their strictness and ease in deriving the joint probability distribution of correlated drought characteristics, thus being appropriate for this analysis.

2.4.2 Bivariate drought frequency analysis

Correct selection of copula type and correct estimation of the dependence parameter are critical steps in constructing any copula model for analysing the drought-related variables of severity and duration. The dependence parameters were estimated in this research based on parametric inference function for margins (IFM) technique involving two steps. It is widely used in hydrology since it provides reliable estimates and is simple to implement. First, marginal distributions with observed data were constructed. The copula dependence parameter identified by the notation θ was thereafter estimated by maximization of the log-likelihood, which is defined for the selected copula as reported by Chukwu [19]

3.0 Results and Discussion

3.1 Drought Quantification and Characterisation

Drought conditions were assessed using the SPI across accumulation of 3, 6, 9 and 12 months. The investigation concluded that the 6-month SPI interval aptly describes the drought behaviour in the basin, giving a better spatial representation than the other time intervals, as illustrated in Table 2. It was observed that, during the course of the present study, the hydrological drought was the most well-demonstrated type. Such a trend can be attributed to soil moisture, which changes within the short to medium time and acts as a primary driver for agricultural drought with lots of changes into different drought categories. In prior works done by, Jimoh *et al.* [23], it was found that 6-month SPI provided the best indication of hydrological drought, whilst agricultural drought was best captured using a 3-month SPI. Figure 2 illustrates the SPI-6 series computed for selected representative stations for the study period and illustrates the changes in drought lengthwise and severity across the basin. From the assessment, cumulative drought severity and duration were thus obtained from the basin with the SPI-6 scale.

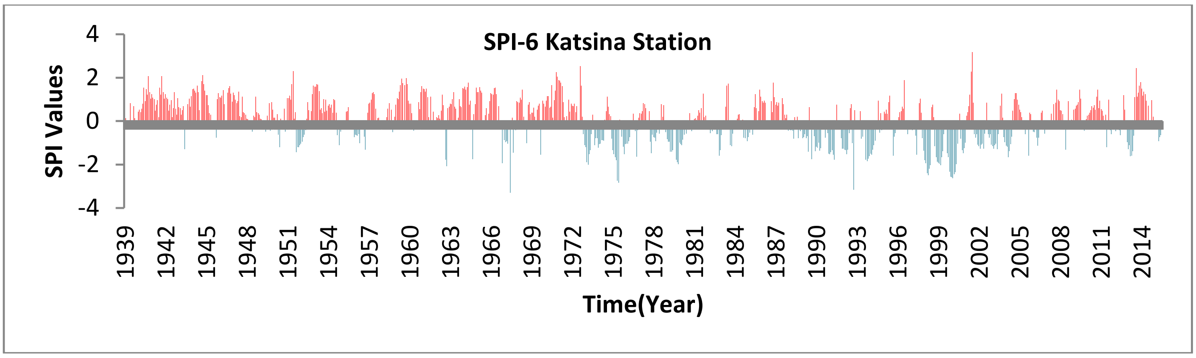

Figure 2: The monthly SPI-6 for the selected gauging station (Katsina)

Table 2: Overview of drought signature in Sokoto-Rima River Basin

SPI	Number of incidence of drought severity					Average
	Gasau	Goronyo	Katsina	Talatamafara	Zobe	
3	16	10	19	2	5	10.4
6	13	10	19	2	9	10.6
9	11	4	13	2	7	5
12	5	7	9	1	2	5

In Figure 3, the association between severity and duration can be seen with the correlation coefficient of 0.7599. The strong correlation was between prolonged periods of drought and high-water deficits (severity). For instance, Gasau station, the drought was the most intense, longest, and produced very high degree of severiy during supposed wet months. Severity recorded were 1.92078 Mm³ in April, 1.88313 Mm³ in May, 1.90999 Mm³ in June, 1.87951 Mm³ in August, and 1.86147 Mm³ in September. This five-month drought worked out a total water deficit of 9.4587 M m³ which gives a vivid snapshot of hydroclimatic characteristics of the basin. This finding supports the fact that higher drought severity is directly associated with the lengthier duration, which is a linear relationship, as shown in Figure 3. In addition, Katsina station, the highest severity ever recorded was 8.4562 Mm³ in 1994 over five months period, while that of Goronyo station was 8.184198 Mm³ over the same period in 1970. Across the basin, mean average drought duration was about 1.82692 months, while average severity was found to be 3.1133 Mm³. This finding further attests to the reliable association between drought quantities and tandem with the submission of Chukwu [24] and, more interestingly, the reliability and effectiveness of the Kendall correlation algorithm for capturing seasonal variation within the basin.

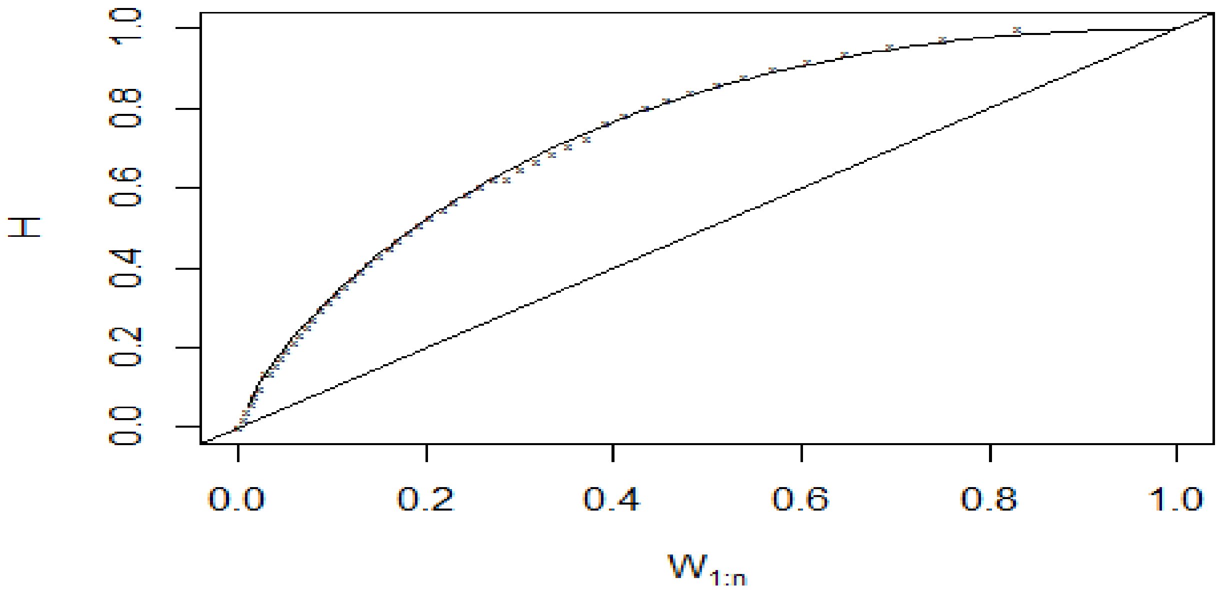

Figure 3: Kendall correlation of drought severity and duration relationship

3.2 Statistical Modelling of Drought Severity and Duration

All the test statistics (Table 3), proved that Generalised Extreme Value (GEV), particularly the Fréchet type was best fitted for the severity series as the test statistics scored less value for GEV among the other tested probability distributions, proved optimal for modelling drought severity in this analysis and for the basin. The graphical illustration of Figure 4 lended credence to the reliability of GEV to model severity. Hence, buttress the efficiency of the GEV model in simulating the observed patterns of drought severity. In the same manner, statistically and visually, Lognormal was identified as the best fit for drought duration as supported by Table 4 and Figure 5. The resulting probability distributions for severity and duration were transformed and mathematically expressed in equation 4 and 5. The computed parameters for GEV are: shape (k)=0.23783663, scale (α)=0.09913426 and location (μ)=0.06003076.
Simultaneously, the lognormal distribution gives mean log (μ)=0.4283828 and standard deviation of the natural logarithm of duration (α)=0.5590378. Equations (18) and (19) are the transformed equation based on the computed statistics that is the cumulative drought severity and duration respectively.

$$f_S(\boldsymbol{s}, 0.0600, 0.0991, 0.2378) = \exp\left\{-\left[1 + 0.2378\left(\frac{(\boldsymbol{S}-0.0600)}{0.0991}\right)^{\frac{-1}{0.2378}}\right]\right\} \quad (4)$$

$$f_D(d) = \frac{1}{2}\left[1 + erf\left(\frac{In(d) - 0.4283828}{0.5590378\sqrt{2}}\right)\right] \quad (5)$$

Table 3: Test Statistics for severity series

Good-of-fit Statistics	Probability Distribution			
	Weibull	Lognormal	Gamma	GEV (Frechet)
Kolmogorov-Smirnov	0.2819	0.2942	0.3048	0.1710
Cramer-Von Mises	0.7255	0.7297	0.7772	0.1657
Anderson-Darling	4.0423	4.0033	4.2865	1.0824
Akaike's Information Criterion (AIC)	207.63	191.69	200.81	145.32

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Good-of-fit Statistics	Probability Distribution			
	Weibull	Lognormal	Gamma	GEV (Frechet)
Bayesian Information Criterion (BIC)	211.53	195.59	204.71	151.177

Table 4: Test statistics for duration series

Good-of-fit Statistics	Probability Distribution			
	Weibull	Lognormal	Gamma	GEV (Frechet)
Kolmogorov-Smirnov	0.3398	0.3744	0.3701	0.3401
Cramer-Von Mises	1.0404	1.1797	1.1482	1.0504
Anderson-Darling	5.7545	6.5858	6.4228	5.9190
Akaike's Information Criterion (AIC)	150.85	135.64	144.24	160.38
Bayesian Information Criterion (BIC)	154.75	139.54	148.14	158.20

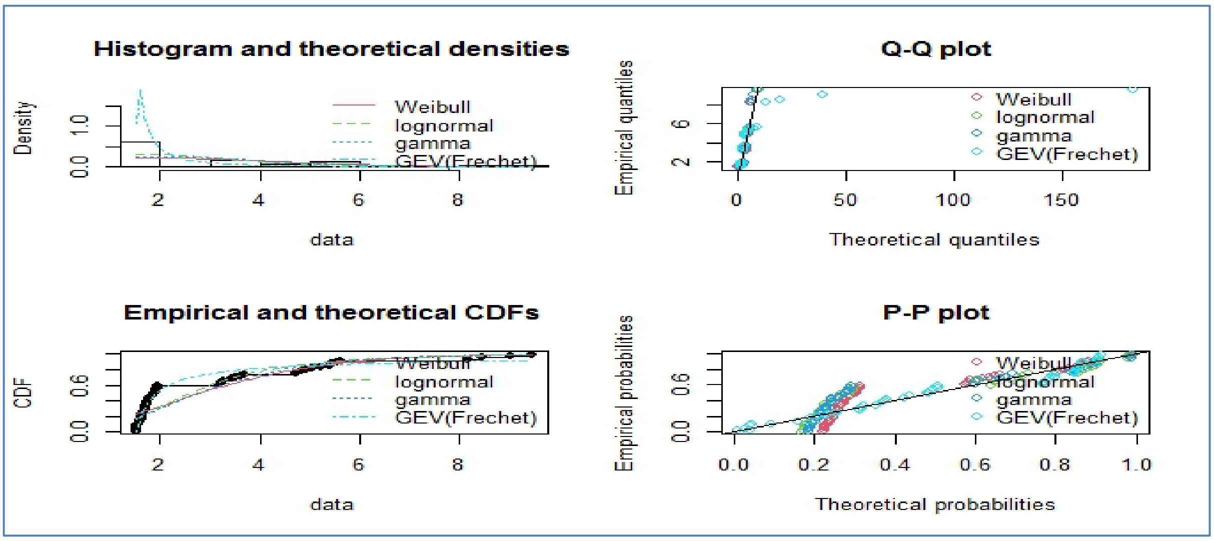


Figure 4: Test statistics plots for probability techniques sampled for severity

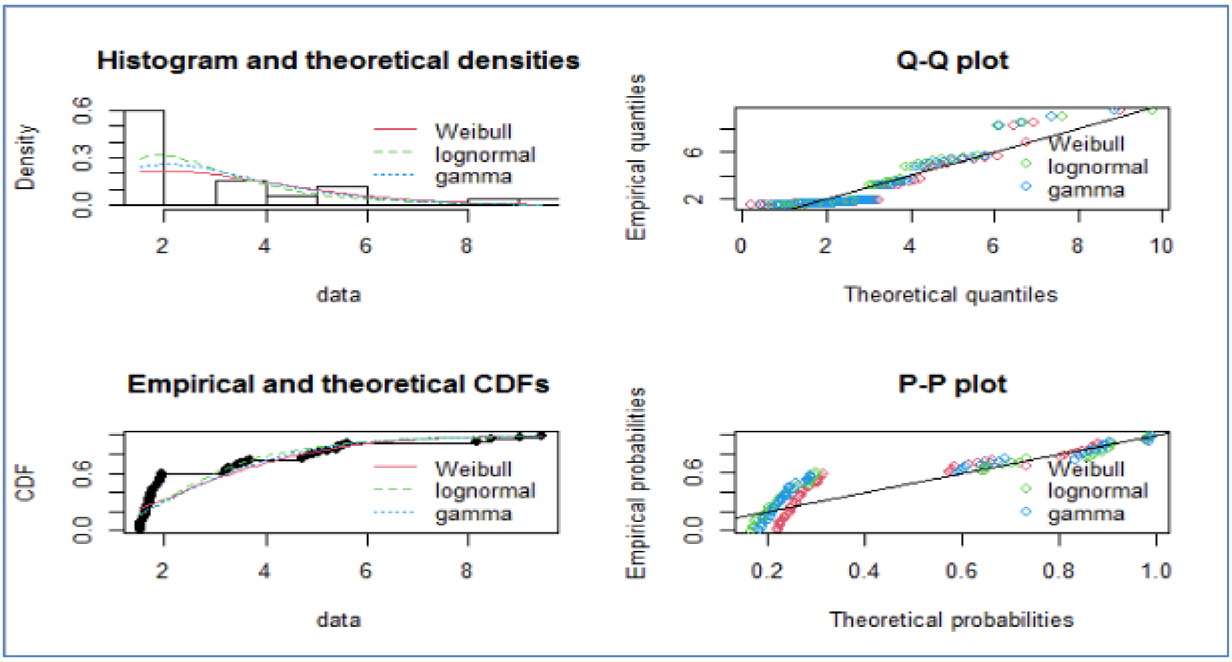


Figure 5: Test statistics plots for probability techniques sampled for duration

3.3 Bivariate copula Modelling and Tail Dependence Analysis

As earlier noted, a suitable model for drought dataset, involves a preliminary investigation of copula families, from correlation analysis drought severity and duration was estimated using Kendall's τ correlation coefficient, which resulted in a value of 0.7599. Since this value do not fall in that range for the Ali-Mikhail-Haq and Farlie-Gumbel-Morgenstern copulas, they were not considered any further. The remaining analyses were hence devoted to the remaining five copulas: Gumbel-Hougaard, Survival, Galambos, Frank, and Clayton. As noted by Chukwu [24] AIC generally performs quite well with smaller data sets, while the SIC/BIC does well with larger sample size. All enhances the credibility of the whole model selection process. The results shown in Table 5 highlight that the Survival (BB8) copula recorded the least RMSE, AIC, and BIC among all copulas tested, indicating its potential to model the dependence between drought severity and duration. In conclusion, the Survival (BB8) copula constitutes the most efficient model for defining the interaction between the two drought features.

As indicated in Table 6 tail dependence coefficients are critical to the selection of suitable copula models. Table 6 records the parametric estimates of the upper tail dependence coefficient for the copulas under consideration, along with their respective values. Furthermore, the upper tail dependence coefficient for drought severity and duration in the area of study, calculated using the CFG estimator, was found to have a value of 0.001. A comparison against Table 6 corroborates that the Survival (BB8) copula falls within the expected bounds and Table 5 shows that survival copula had less value for all the test statistic employed. Therefore, formulation of the modified Survival (BB8) copula results in bivariate drought distributions represented in Equation (5).

$$S(d,s) = 1 - F_D(d) - F_S(s) + F(d,s) \quad (5)$$

Equation 5 represented ideal combinations of drought duration and severity by the best fitted copula, it suffices to note, that the drought severity and duration in the studied area was effective described or mimicked via Survival (BB8) copula. This claim was further strength by contour line (Figure 7) showing a strong relationship between severity and duration and a stepwise increment.

Table 5: Test statistics for bivariate copula distribution

Copula	Parameter (α, β)	Log-likelihood function	RMSE	AIC	BIC
Clayton	6.331	0.7334	0.0749	-286.6479	-284.6967
Frank	11.370	34.100	0.0300	-247.1391	245.1380
Galambos	5.802	2.1475	0.0450	-297.4314	-295.4802
Gumbel	3.511	2.1475	0.0429	-313.1866	-311.2354
Survival (BB)	6.000, 0.970	43.4938	0.0023	-82.9876	-79.0850

Table 6: Estimated parameters and the corresponding upper tail dependence coefficient values

Copula	λ_u	Value
Clayton	0	0
Frank	0	0
Galambos	$2^{-1/\theta}$	0.8874
Gumbel	$2\text{-}2^{1/\theta}$	0.7818
Survival (BB8)	0	0

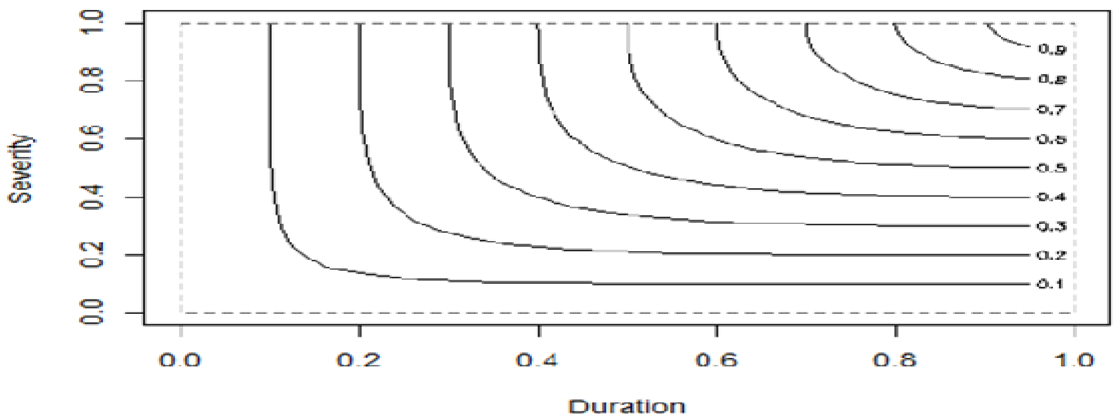


Figure 6: illustration of the joint probability of duration and severity

3.4 Bivariate probability assessment of drought event

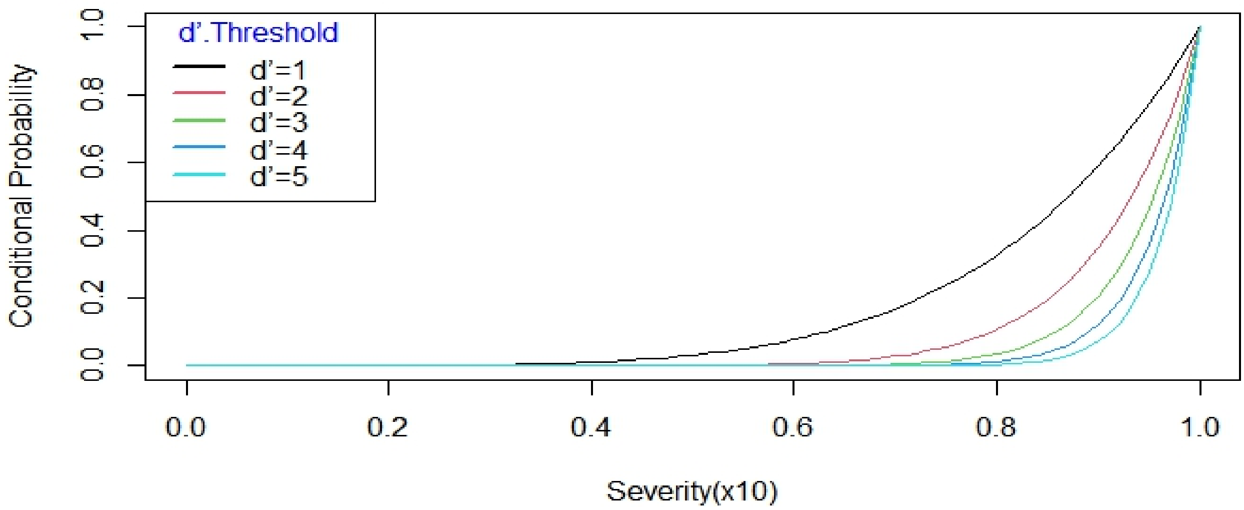
In line with Shiau and Shen (2001) drought probability was appraised with equation (6 and 7).

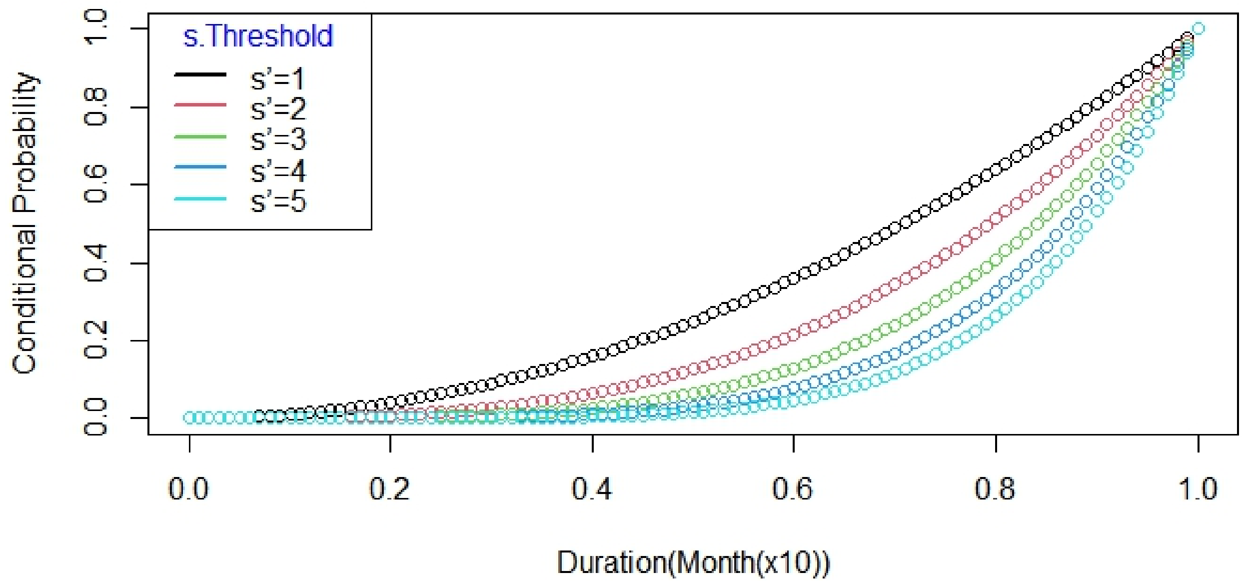
$$P(S \leq s | D \geq d') = \frac{F_S(s) - C(F_D(d), F_S(s))}{1 - F_D(d')} \quad (6)$$

$$P(D \leq d | S \geq s') = \frac{F_D(d) - C(F_D(d), F_S(s'))}{1 - F_S(s')} \quad (7)$$

Figures 8a and 8b illustrate the conditional distribution of drought duration, given that severity exceeds certain thresholds (equation 6), and the conditional distribution of drought severity when duration exceeds certain thresholds (equation 7), respectively. For the purpose of analysing drought episodes in different severities and durations, a conditional probability curve was established to represent the association between drought durations, severities and thresholds. Drought occurrences are analysed with threshold levels of 1, 2, 3, 4, and 5, as espoused in Figures 8a and 8b. The condition distribution for drought duration concerning a specified severity appears in Figure 8b, while the common figure-related conditional drought severity in the specific duration is illustrated in Figure 8a. All Figures indicated that as drought duration increased, the related probability distribution of severity somewhat decreased. These curves are efficient in determining the probability of exceeding a certain severity corresponding to duration (Figure 8a) or surpassing a specific duration for a given severity (Figure 8b). For instance, Figure 8a reports a 0.01 probability that the drought severity is circumvented by 6 Mm^3 above the one-month threshold for drought duration. Similarly, probability of 0.178 was observed when the threshold remains 8 Mm^3 with the same one-month condition. Alternatively, Figure 8b portended, the condition when drought severity is beyond 1 Mm^3, the odds of a drought lasting fewer than six months would be 0.175, while the probability of duration being less than eight months under the same condition was 0.401.

(a)





(b)

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Figure 8: Conditional Probability Plots of (a) drought Severity given drought duration above a certain Limit and (b) drought duration given drought severity above a certain Limit.

3.5 Formulation of Drought Severity-Duration-Frequency Curves

In line with Chukwu [24] employed statistical models that predicts the likelihood of droughts exceeding set thresholds in terms of duration or severity taken into account the expected frequency of occurrence, cumulative drought duration, and severity distribution functions as illustrated in equation 8 and 9 respectively.

$$T_D = \frac{E(L)}{1-F_D(d)} \tag{8}$$

$$T_S = \frac{E(L)}{1-F_S(s)} \tag{9}$$

T_D indicates the return time for the drought duration that equals or exceeds a specified Threshold. T_S indicates the return period of a drought severity greater than or equal to a certain value. Furthermore, L represents the inter-arrival time, between the end of one drought and the beginning of another. Expected inter-arrival time of droughts is denoted as E(L).

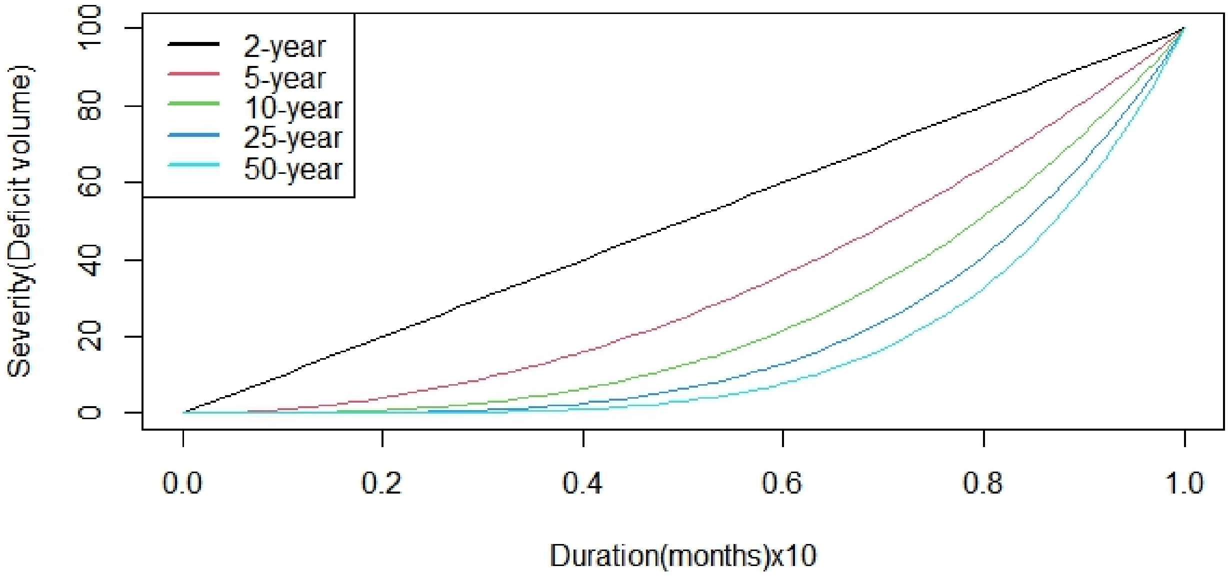

Figure 9: Severity-Duration-Frequency curves at various return periods

According to the drought series, it was estimated that the average interval between the successive occurrences of droughts was nearly 0.9875 months. Among the observational stations considered; Gusau had the most prolonged and severe drought conditions for the whole duration during supposed rainy season. In particular, the severity values for the months of April, May, June, August, and September were 1.92078, 1.88313, 1.90999, 1.87951, and 1.86147 Mm^3, respectively. According to Figure 9, a severe drought event having average deficit volume of 1.890958 Mm^3 and lasts for about five months and may likely to recur at least after every 25th year from 1973 at Gusau station. Similar to that, the worst drought episode was recorded in Katsina in 1994, whereby the highest severity of 8.4562 Mm^3 was recorded over five months duration, while Goronyo recorded a drought severity of 8.184198 Mm^3 over the same duration in 1970. The above findings imply that both stations would expect a similar drought event with a recurrence interval hoves around five years, as observed in Figure 9. The average drought severity and duration was 1.82692 Mm^3 and 3.1133 months at the entire basin. This means that drought events in the region are likely to variegate in their different characteristics and will have an approximate period of recurrence about every five years. The findings of the study also implied a corresponding expected level of drought conditions throughout the wider Savanna region where this study area is located.

4.0 Conclusions

This study comprehensively examined the drought characteristics of the Sokoto-Rima River Basin using a copula-based modelling framework to establish the severity-duration-frequency (SDF) relationships. The Standardised Precipitation Index (SPI), evaluated across multiple timescales, identified SPI-6 as the most effective metric for capturing hydrological drought events within the basin. Marginal distributions were fitted to drought

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severity and duration, with the Generalised Extreme Value (GEV–Fréchet) and Lognormal distributions emerging as the best fits, respectively. Copula functions were employed to model the interdependence structure between severity and duration, with the Survival BB8 Copula outperforming other families in terms of goodness-of-fit metrics such as AIC, BIC, and RMSE. The strength of dependency, confirmed by a Kendall's tau of 0.76, emphasised the need for joint probabilistic modelling of drought attributes. The derived SDF curves provide a practical tool for estimating the likelihood and intensity of drought events, which is crucial for designing resilient water resource systems, particularly in a region prone to frequent hydroclimatic extremes. These findings underscore the value of copula-based approaches in enhancing drought risk assessment and offer actionable insights for hydrological planning and early warning systems in semi-arid environments.

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