



High-Fidelity Wake Modeling for Multi-Turbine Optimization in Complex Terrain: A Review

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Abstract

Wake interactions between turbines are a primary constraint on wind farm energy yield and structural reliability, and this constraint is magnified in complex terrain where topography drives speed-up, channelling, and flow separation that classical wake models were never designed to capture. This paper reviews the state of the art in wind turbine wake modelling and multi-turbine optimization, tracing the field's progression from simplified engineering models (Jensen, Gaussian) through high-fidelity large eddy simulation (LES), toward multi-fidelity and data-driven surrogate frameworks. The review synthesizes findings on wake fundamentals, wake-wake interactions, wake steering as a control strategy, LES-based simulation, multi-fidelity and surrogate modelling, terrain and atmospheric-stability effects, and optimization strategies balancing energy yield against structural loads. A synthesis table condenses the methods, findings, and limitations of representative studies published between 2020 and 2025. The review concludes by identifying a persistent gap: the absence of a scalable, high-fidelity wake model integrated directly into a multi-turbine optimization framework for complex terrain, and outlines how closing this gap can improve energy capture, extend turbine service life, and reduce the levelized cost of energy.

Keywords: Wind turbine wakes, wake steering, large eddy simulation, complex terrain, multi-fidelity modelling, surrogate modelling, wind farm optimization, levelized cost of energy.

1.0 Introduction

Wind energy is central to the global transition toward low-carbon electricity, yet the performance of a wind farm is rarely the sum of its turbines' individual capabilities. Downstream machines operate in the momentum-deficient, turbulence-enriched wakes of upstream rotors, which depress energy yield and accelerate fatigue damage [1]. In poorly optimized layouts, wake losses can reduce farm output by as much as 20-30% [1]. The scale of this problem has motivated a large and fast-moving body of research spanning fluid dynamics, control engineering, and machine learning, recently synthesized into a set of grand-challenge priorities for the field that explicitly call for stability-aware models, sensor fusion, and multi-fidelity frameworks [3].

Complex terrain compounds the difficulty. Hilly, forested, and coastal sites are increasingly attractive for development because of their strong wind resources, yet classical engineering wake models assume flat terrain and struggle to generalize once topography introduces speed-up, channeling, and flow separation [1]. Multi-fidelity frameworks that combine fast wake models with large eddy simulation (LES) have begun to close this gap, demonstrating improved power output and reduced wake losses by using LES to correct low-order models within Bayesian optimization loops [2]. Modern turbines, now exceeding 100 m in height with blades longer than a football field [4], make the stakes of getting wake prediction right progressively higher: even small percentage errors in yield forecasts translate into large absolute impacts on project economics and grid planning [5], [6], [7].

This paper reviews the wake-modeling and multi-turbine optimization literature relevant to complex-terrain wind farms. Section 2.0 establishes wake fundamentals; Section 3.0 examines wake-wake interactions; Section 4.0 reviews wake steering as a control strategy; Section 5.0 discusses LES; Section 6.0 covers multi-fidelity and surrogate modeling; Section 7.0 addresses terrain and atmospheric-stability effects; Section 8.0 reviews optimization strategies; Section 9.0 synthesizes representative studies in tabular form; and Section 10.0 identifies the research gap that motivates continued work in this area.

2.0 Wind Turbine Wake Fundamentals

A wind turbine converts the kinetic energy of moving air into electricity by extracting momentum from the flow, leaving behind a velocity deficit and elevated turbulence: the wake. The wake is conventionally divided into a near-wake region, dominated by strong shear layers and coherent vortex structures shed from the blades and nacelle, and a far-wake region, where turbulent mixing gradually restores wind speed while increasing variability in the inflow experienced by downstream turbines [1]. In-situ measurements using unmanned aerial vehicles have confirmed this two-region structure and its sensitivity to ambient conditions [8].

Wake recovery is strongly modulated by atmospheric stability: stable stratification suppresses vertical mixing and prolongs the velocity deficit, whereas unstable conditions enhance turbulence and accelerate recovery. High-fidelity LES studies have further shown that wakes are not static but meander—a low-frequency oscillation of the wake centerline driven by large-scale atmospheric boundary-layer turbulence [9]. Wake meandering raises the variability of downstream inflow and, in turn, the fatigue loading experienced by downstream rotors. Field-synchronized actuator-line LES studies confirm that large-scale vortices and meandering dominate far-wake behavior in utility-scale turbines, providing improved parameterizations of wake-added turbulence [11]. At the farm scale, overlapping wakes from multiple turbines compound these effects, amplifying turbulence intensity and reducing overall efficiency [10].

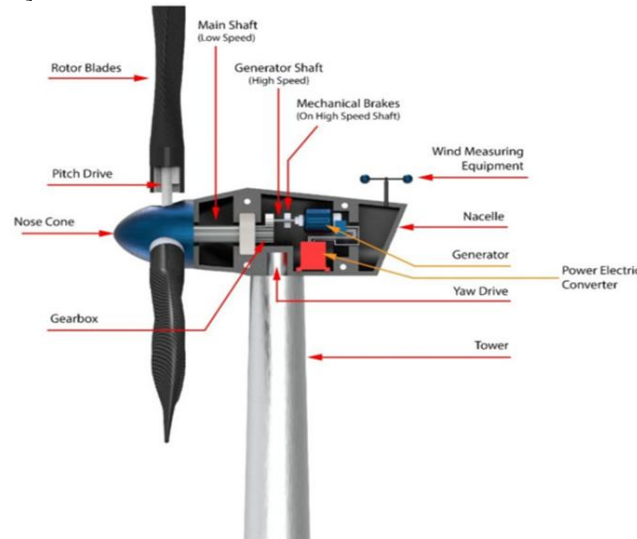


Figure 1: Components of WECS (Chaudhuri *et al.*, 2021)

3.0 Wake Interactions in Wind Farms

When turbines are arranged in rows or clusters, their individual wakes overlap, merge, and deflect, producing spatially varying power deficits and load hotspots that are difficult to predict from single-wake physics alone. Three mechanisms dominate: wake-wake superposition, in which overlapping deficits combine nonlinearly and simple additive models fail to capture the resulting turbulence structure [10]; wake deflection, in which ambient shear, terrain-induced curvature, or yaw misalignment shifts the wake laterally or vertically [14]; and wake impingement, in which a wake directly strikes a downstream rotor, causing sharp drops in inflow velocity and spikes in turbulence intensity.

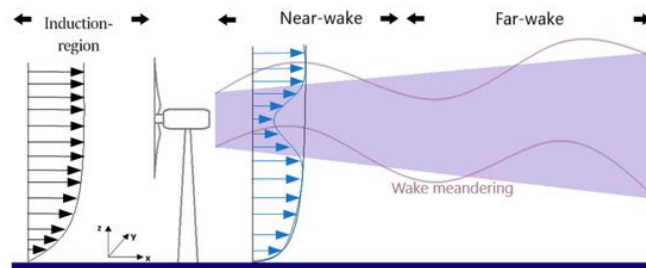


Figure 2: Schematic representation of the three wake regions and of the wake meandering phenomenon, which is indicated by the curved lines. The wind profiles upstream (in black) and downstream (in blue) from the turbine are shown (Alaoui-Sosse *et al.*, 2022)

Refined superposition formulations that enforce momentum conservation across overlapping wakes have been shown to improve multi-wake predictions relative to simple linear or energy-based superposition [12]. Cumulative interactions between blockage effects upstream of a farm and wake effects downstream have also been observed and modeled at the farm scale, underscoring that wake interactions extend beyond the immediate rotor-to-rotor scale [13]. These compounding effects make layout optimization a genuinely multi-scale problem, since a turbine's performance depends jointly on its neighbors' positions, the ambient wind direction, and the prevailing stability regime.

4.0 Wake Steering as a Control Strategy

Wake steering intentionally yaws upstream turbines to deflect their wakes away from downstream rotors, trading a small power loss at the yawed turbine for a net farm-level gain. Early closed-loop field trials combining

Gaussian wake models with SCADA and lidar data demonstrated directional power gains of 2-8% with minimal load penalties [15], and subsequent multi-site trials incorporating explicit load constraints reported 3-5% energy gains in targeted wind directions while keeping structural loading within acceptable bounds [16].

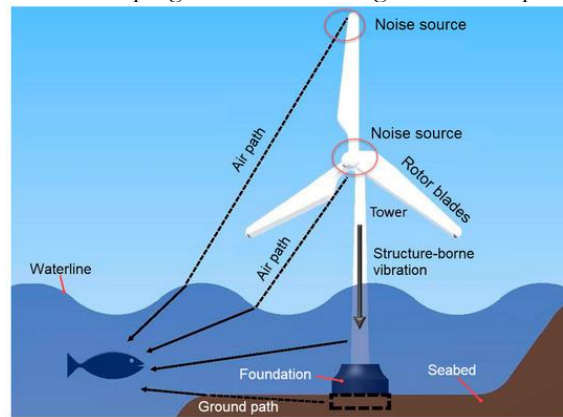


Figure 3: Sound transmission path of an offshore wind turbine (Bošnjaković, 2023)

The effectiveness of wake steering depends on several interacting factors. Accurate, low-latency estimation of wind direction is critical, since misaligned yaw commands can worsen wake overlap rather than relieve it; incorporating preview information from met masts and nacelle sensors has been shown to improve direction tracking and reduce misalignment errors relative to purely reactive control [17]. Lidar-assisted preview control extends this idea by anticipating upcoming inflow changes before they arrive at the rotor [18]. Yaw-based multi-objective formulations that jointly consider power gain and mechanical stress provide a further layer of refinement over single-objective steering schedules [19], and model predictive control strategies that incorporate direction-dependent constraints and stability indicators have outperformed static steering schedules under realistic operational uncertainty [20].

5.0 Large Eddy Simulation in Wind Energy

Large eddy simulation resolves the large, energy-containing turbulent eddies directly while modeling smaller subgrid-scale motions, giving it a level of physical realism that Reynolds-Averaged Navier-Stokes (RANS) approaches cannot match for unsteady phenomena such as wake meandering, shear-layer roll-up, and vortex shedding [1]. Comparative studies of actuator-disk and actuator-line rotor representations within LES show that actuator-line models better capture near-wake structure, while actuator-disk models remain acceptable and considerably cheaper for far-wake trends, making the choice between them dependent on the modeling objective [21].

In complex terrain specifically, LES has been used alongside immersed-boundary methods to resolve flow separation, channeling, and localized turbulence hotspots over ridges and forested slopes with a fidelity that RANS cannot reach, though at substantially higher computational cost; a hybrid workflow using RANS for preliminary screening and LES for detailed site assessment has been proposed as a practical compromise [22], [23]. Validation remains essential: turbulence spectra generated by high-fidelity wind-field simulators have been benchmarked directly against theoretical turbulence spectra, with close agreement between simulated and theoretical spectra lending confidence to LES-based turbulence characterization for turbine design [24].

6.0 Multi-Fidelity and Surrogate Modeling

Because LES remains too expensive to embed directly inside iterative optimization loops, multi-fidelity modeling has emerged as the dominant strategy for reconciling accuracy with computational tractability. The approach uses fast, low-order models for broad exploration of a design or control space, then selectively applies expensive high-fidelity evaluations to correct bias and quantify uncertainty—commonly formalized through a co-kriging relationship in which the high-fidelity response is expressed as a scaled low-fidelity response plus a Gaussian-process discrepancy term. Multi-fidelity Bayesian optimization built on this idea has demonstrated robust power gains in wake steering under a constrained LES sampling budget [2], and a hierarchical variant that uses cost-aware acquisition functions to guide LES sampling has produced higher annual energy production than single-fidelity baselines while querying LES far less frequently [25].

Surrogate models extend this idea by learning direct approximations of wake velocity fields, turbine loads, or farm power. Gaussian-process surrogates that embed physical conservation laws into their kernel functions achieve lower extrapolation error and better-calibrated uncertainty than standard Gaussian processes when trained on LES data [26], and related stability-aware Gaussian-process surrogates have been used to predict farm power output directly from atmospheric state [27]. Physics-informed surrogates trained on multi-fidelity datasets that

blend LES and field data have been shown to generalize across nonuniform atmospheric conditions [28], while physics-informed neural networks that embed the governing PDE residuals directly into the training loss achieve lower error than purely data-driven networks in sparse-data regimes [29]. For load forecasting under uncertainty, mixture density networks trained on LES and field subsets output full probability distributions of turbine loads, enabling damage-equivalent-load estimates with explicit uncertainty bounds rather than single-point predictions [30], [31]. Most recently, multi-fidelity dynamic wake mapping built on an attention U-Net architecture has shown that both direct-mapping and transfer-learning strategies can reconcile low- and high-fidelity dynamic wake fields while substantially reducing the high-fidelity training data required, underscoring the continued rapid pace of progress in this area [45].

7.0 Complex Terrain and Atmospheric Stability Effects

Complex terrain modifies both inflow and wake recovery through speed-up over ridges, channeling through valleys, and enhanced turbulence over forested or rough slopes. Coupled mesoscale-microscale simulations that incorporate heterogeneous roughness and canopy effects have shown that terrain-stability interactions strongly shape wake deflection and recovery, often invalidating the flat-terrain assumptions built into classical engineering models [32]. At a larger scale, large wind farms have themselves been shown, through coupled mesoscale-LES frameworks, to generate atmospheric gravity waves that propagate beyond the farm boundary and feed back on inflow conditions, turbulence intensity, and wake recovery [33], [34]—a reminder that farm-atmosphere coupling is not confined to the immediate vicinity of the turbines. Field-validated large-eddy simulations of hilly terrain confirm that terrain-induced pressure gradients deflect the wake and produce an asymmetric velocity recovery that simple superposition of a flat-terrain wake onto the terrain wind field cannot reproduce, motivating a dedicated analytical wake model for complex topography [46]. Complementary field and simulation work on a real, operating wind farm has quantified this effect directly, showing that positive streamwise terrain slopes shorten wake-recovery length relative to descending terrain, with recovery lengths ranging from roughly 6D on favorable slopes to over 9D on unfavorable ones [47].

Atmospheric stability itself, quantified through the Monin-Obukhov length, governs how quickly wakes recover: stable conditions suppress mixing and prolong wake deficits, while unstable conditions enhance turbulence and accelerate recovery. LES studies of stratified inflow across large arrays have found that aligned turbine layouts are markedly more sensitive to stability than staggered layouts, motivating regime-aware spacing and control strategies rather than a single fixed layout for all conditions [35]. Engineering-model refinements that introduce stability-dependent parameters for shear-layer growth and added turbulence, calibrated against LES and mast data across multiple stratification regimes, have improved wake-deficit and turbulence predictions over baseline Gaussian models while remaining computationally tractable for planning and control [36].

8.0 Optimization Strategies for Wind Farm Design and Control

Wind farm optimization balances energy yield against structural loads, typically posed as maximizing the sum of turbine power output minus a weighted load penalty over yaw angles and turbine placement. Co-optimizing layout and control together, rather than sequentially, has been shown to reduce wake overlap and increase annual energy production more effectively than a two-stage design process [37]. Multi-objective formulations solved with evolutionary algorithms reveal clear, direction-dependent trade-offs between energy capture and load across competing layout and yaw strategies, supporting risk-aware planning rather than a single 'optimal' configuration [38]. Reinforcement-learning controllers trained on high-fidelity simulators have achieved power and load performance competitive with heuristic control while adapting more readily to inflow variability, though transferring policies from simulation to real turbines remains an open challenge [39].

Load mitigation is commonly quantified through the damage-equivalent load, and strategies such as load-constrained wake steering, derating, and individual pitch control have been used to manage fatigue while preserving power gains, with load-mitigation control designs dating back to path-of-vortices operation and continuing to inform current practice [40]. Benchmarking of updated momentum-consistent Gaussian models against multi-season SCADA data shows they consistently outperform older Jensen-type models in predicting both power deficits and turbulence intensity, supporting modernization of the operational models still used across much of the industry [43], and complementary work refining turbulence-intensity and entrainment-rate parameterizations against mast and SCADA data has further improved downstream turbulence and load forecasts [44]. Because wake losses and elevated loads both feed directly into higher operations-and-maintenance costs and reduced annual energy production, physics-based wake mitigation has been linked quantitatively to leveled cost of energy and project bankability, giving financial weight to modeling improvements that might otherwise be framed purely in technical terms [41]. Layout studies specifically targeting offshore arrays under turbulent inflow reinforce that spacing and orientation decisions measurably shape farm-wide efficiency and wake characteristics [42].

9.0 Synthesis of Representative Studies

Table 1 condenses representative studies discussed above, organized by primary method, principal finding, and the main limitation identified by the original authors. The synthesis illustrates the field's trajectory from fast-but-simplified engineering models toward physically grounded, computationally tractable hybrids.

Table 1: Synthesis of representative wake-modeling and optimization studies

Study	Method	Principal Finding	Limitation
Porté-Agel et al. [1]	Comparative review: Gaussian, RANS, LES	Gaussian models are fast but miss stability/terrain effects; LES is accurate but costly	Uneven field-data availability; inconsistent validation protocols
Mole & Laizet [2]	Multi-fidelity Bayesian optimization	LES-corrected low-order models improve steering power gains	Transferability across sites; surrogate calibration dependence
Stieren & Stevens [10]	LES, multiple turbine rows	Heterogeneous power deficits, elevated turbulence in downstream rows	Computationally intensive; idealized boundary conditions
Fleming et al. [16]	Field trials, SCADA/strain-gauge	3–5% energy gains with controlled load increases	Direction dependence; seasonal variability
Martínez-Tossas & Purohit [21]	Actuator-disk vs. actuator-line LES	Actuator-line better resolves near-wake; actuator-disk cheaper for far-wake	Limited inflow variability tested
Vali et al. [26]	Physics-informed GP surrogate	Lower extrapolation error, better uncertainty quantification	Complex physics-consistent kernel design; training cost
Zhan et al. [29]	Physics-informed neural network	Lower error in sparse-data regimes; improved generalization	Training instability; hyperparameter sensitivity
Abkar & Porté-Agel [35]	LES, stratified inflow, large arrays	Aligned layouts more stability-sensitive than staggered	Periodic-domain assumptions; simplified terrain
Pedersen & Larsen [37]	Joint layout/control optimization	Co-optimization outperforms sequential design	Simplified turbulence parameterization
Housner & Hernando [41]	Engineering optimization + LCOE model	Wake mitigation lowers O&M cost, improves IRR	Uncertain long-term stability statistics and financial assumptions

10.0 Research Gap and Future Directions

Despite the substantial progress catalogued above, no single framework yet integrates a scalable, high-fidelity wake model directly into a multi-turbine optimization pipeline for complex terrain. High-fidelity methods such as LES remain accurate but rarely scale to full-farm optimization; data-driven surrogates accelerate prediction but often fail to generalize across terrains and atmospheric regimes when trained on narrow datasets [28]; and

optimization frameworks frequently neglect the uncertainty introduced by terrain-induced turbulence altogether, which can lead to layouts that look optimal on paper but underperform on site [38].

Closing this gap calls for four coordinated advances: (i) LES combined with reduced-order modeling to capture terrain-induced nonlinear wake dynamics without prohibitive cost; (ii) multi-fidelity surrogate frameworks that blend LES data with analytical wake models to make high-fidelity information usable inside iterative optimization; (iii) machine-learning predictive models trained on multi-fidelity datasets to generalize across terrains and wind regimes and quantify uncertainty; and (iv) multi-objective optimization algorithms, such as particle swarm optimization and genetic algorithms, that jointly optimize yaw control, spacing, and layout under realistic terrain and stability variability. Bringing these four elements together in one unified methodology—rather than pursuing them as separate publications—would advance both the academic understanding of terrain-aware wake dynamics and the practical tools available to wind farm developers and policymakers designing renewable energy systems in challenging terrain [3], [41].

11.0 Conclusion

Wake modeling has progressed markedly since the era of purely analytical engineering models, moving through high-fidelity LES and into multi-fidelity, physics-informed, and data-driven surrogate approaches capable of supporting real-time or near-real-time decision-making. In complex terrain, however, these advances remain fragmented: accuracy, computational efficiency, uncertainty quantification, and multi-objective optimization have each been demonstrated individually but rarely combined within a single validated framework. A unified methodology that integrates LES-informed reduced-order modeling, multi-fidelity surrogates, terrain- and stability-aware machine learning, and multi-objective turbine coordination offers a credible path to improved energy yield, extended turbine lifespan, and lower levelized cost of energy—outcomes that matter both academically and for the continued deployment of wind power in topographically challenging regions.

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