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Development of Energy-Aware Hamiltonian-Based Fixed-Time Chaos Control and Synchronisation of Brushless DC Motor for Electric Vehicle Application

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Abstract

Electric Vehicle (EV) Drivetrains commonly employ in electric vehicles, and Brushless DC (BLDC) motors, which are widely used for their efficiency, power density, and ease of maintenance. Their architecture, however, contains one particular fault in its operation: under certain voltage, torque, and damping conditions, they can exhibit deterministic chaos, caused by the nonlinear coupling between their electrical and mechanical states. This poses a real danger in EVs, as battery fluctuations, thermal drift and sudden changes in torque during acceleration or regenerative braking can readily drive the motor into unpredictable speed and torque oscillations, a problem that current control methods struggle to solve. Although a direct control law is not obtained from Hamiltonian energy analysis alone, such analysis helps explain the origins of this chaos. In contrast to fractional-order, fuzzy, and neural controllers, which offer only bounded or exponential convergence guarantees a critical drawback during operational faults an energy-aware, fixed-time controller based on the Hamiltonian method is proposed to suppress chaos and synchronise BLDC drives. A dimensionless chaos model is first developed, and its energy dynamics are examined to isolate the exact imbalance between energy injection and dissipation that causes chaotic behaviour. The error dynamics are then reformulated into a port-controlled Hamiltonian system. This structure enables a fixed-time control law that directly regulates the Hamiltonian gradient without requiring high-gain switching. A Lyapunov proof guarantees a strict settling-time bound defined independently of the motor's initial state. All simulations were implemented and validated in MATLAB/Simulink, with controller performance verified against the theoretical settling-time bound obtained from the Lyapunov analysis. Finally, simulation results for reference tracking and master-slave synchronisation verified the controller's performance. By unifying energy-based chaos analysis with fixed-time stability theory, this framework provides a physically interpretable control mechanism ideal for safety-critical EV applications such as torque vectoring and anti-lock braking.

Keywords: *Brushless DC motor; Deterministic chaos; Electric vehicle; Fixed-time stability; Hamiltonian energy.*

1.0 Introduction

Brushless DC (BLDC) motors are widely used in modern electric vehicle drivetrains. These motors offer high efficiency, a favourable power-to-weight ratio, low maintenance, and electronic commutation in place of a mechanical brush assembly. In addition to traction drives, BLDC drives are increasingly found in auxiliary drives and regenerative braking assemblies, and in more demanding applications such as torque vectoring and anti-lock braking systems. As the tolerances and dynamics of BLDC operation are pushed ever tighter, the electromechanical performance of the drive itself has naturally come under scrutiny.

This scrutiny has brought a more unusual aspect of the technology to the surface. The governing equations of a BLDC motor are a set of nonlinear, multiplicative equations, which relate the electrical state, represented by stator currents and back-electromotive force, to the mechanical state, represented mostly by the rotor angular velocity. This coupling can generate true deterministic chaos: bounded but non-periodic motion that is extremely sensitive to small changes in initial conditions, under specific conditions of supply voltage, load torque and damping [1], [2]. This is not just a simulation plaything. In an actual EV, battery voltage drops under load, ambient and internal temperatures change over a drive cycle and torque demand increases and decreases rapidly during acceleration, braking and regenerative events that are known to cause a BLDC drive to switch from stable operation to chaotic torque and speed oscillation [3]. Left unaddressed, the consequences are torque ripple, additional acoustic noise, accelerated mechanical fatigue, and, in safety-relevant functions, a genuinely unpredictable response at precisely the moment predictability matters most.

Research on chaos in BLDC drives has advanced along several parallel tracks. One strand has focused on characterising the phenomenon itself. In [1]showed that chaos could be induced through current time-delayed feedback, while [2]extended the analysis to a fractional-order BLDC model and demonstrated that the fractional derivative order behaves as a bifurcation parameter in its own right. Faradja and Qi, across a sequence of studies

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[4],[5],[6], mapped the local bifurcation structure of the canonical BLDC chaotic model in detail and went on to identify multi-stability, hidden chaos, and transient chaotic regime dynamics that [7] later linked more broadly to offset-boosting behaviour in Hamiltonian conservative chaotic systems. More recently, In [8] confirmed spiking and multi-stable dynamics in a non-smooth-air-gap BLDC configuration through circuit implementation, [3] extended the analysis to a heat-affected drive under load torque, and [9] derived semi-analytical solutions and stability boundaries for the underlying nonlinear system. Taken together, this body of work leaves little room to argue that BLDC chaos is a modelling artefact; it is a robust, physically observable phenomenon.

A second track has pursued control and suppression directly. Fractional-order and fractal-fractional formulations feature prominently, largely because they capture memory effects that integer-order models cannot [10], [11], [2]. A related but distinct objective is synchronisation, forcing a slave drive to track a chaotic or otherwise unpredictable master trajectory, which has been explored for EV traction applications [12], for sensor-less configurations that avoid additional instrumentation cost [13], and through adaptive fuzzy and neural architectures [14], [15], [16] has since offered a comparative taxonomy of these synchronisation strategies. A third, smaller track treats the problem through energy and Hamiltonian formalisms: and [17] introduced an energy-cycle decomposition of the BLDC chaotic system,[18] generalised this to energy- and volume-conservative chaotic systems more broadly, and [6] applied a Hamiltonian-based energy analysis specifically to the BLDC case. Outside the BLDC literature, fixed-time stability tools including a Gaussian-error-function-based theorem applied to Lorenz-system synchronisation [19] and finite/fixed-time results for permanent magnet synchronous motors under noise [20] have matured into a transferable analytical toolkit that this domain has only begun to draw on. Diagnostic work by the authors in [21] and [22], including hardware-validated results on drone propulsion systems, together with application-oriented excitation-strategy selection by [23], confirms that chaos-theoretic tools in this domain now extend well beyond suppression alone.

Despite this progress, the energy-based and fixed-time control strands of the literature have developed largely in parallel rather than together. Hamiltonian and energy-cycle analyses [6], and [18], [17] have served chiefly as a lens for understanding how chaos arises, rather than as the foundation for a control law that acts directly on the drive's energy structure. Conversely, the control-oriented literature on fractional-order sliding-mode designs, adaptive fuzzy and neural schemes, and genetic-algorithm-tuned controllers [10], [15] [3], [2] has relied largely on asymptotic or finite-time convergence results, in which the settling time either grows without bound or depends on the initial condition. Given that a BLDC drive in an EV can enter its chaotic regime from essentially any operating point, an initial-condition-dependent recovery time is a poor fit for safety-critical torque and speed regulation. Fixed-time stability theory, which guarantees a convergence bound independent of the initial state, has been demonstrated for related chaotic systems the Lorenz system [19] and the PMSM under noise [20], [24] but has not, to date, been formulated for a BLDC drive in a way that is simultaneously energy-aware, that is, derived from and acting on the system's own Hamiltonian structure rather than treating the plant as a generic nonlinear system. No study identified in the reviewed literature combines a port-controlled Hamiltonian formulation of the BLDC chaotic system with a fixed-time convergence guarantee applied specifically to chaos suppression and synchronisation in an EV context.

This leaves a concrete gap. Existing BLDC chaos controllers tend to fall into one of two categories: those that offer strong physical insight into the drive's energy behaviour without a hard convergence guarantee, and those that offer a convergence guarantee unconnected to the drive's underlying energy structure, with a settling time that still depends on initial conditions. For an EV powertrain, where the drive may need to recover from a chaotic excursion triggered by a sudden load or voltage transient at any point during operation, this is a meaningful shortfall rather than a theoretical nicety. The problem addressed in this study is therefore twofold: first, how to formulate the BLDC drive's chaotic dynamics in port-controlled Hamiltonian form suitable for energy-based control design; and second, how to derive from that formulation a control law that suppresses chaos and enforces master–slave synchronisation within a settling time fixed in advance, bounded, and independent of the drive's initial state.

This research work has three levels of contribution. It theoretically extends the fixed-time stability analysis previously demonstrated on the Lorenz system and synchronisation of PMSM under noise into a Hamiltonian framework, in which the port is controlled, specific to the BLDC chaotic model and represents a link between the energy-analytic tradition, established by [17] and [6], and the convergence-guarantee tradition, established in the literature for fixed-time control. Conceptually, it generates a controller that has a physical interpretation in a real sense it operates on the same energy parameters that cause the drive's chaotic dynamics, instead of regarding the motor as a nonlinear system to be dominated by high-gain switching terms, which is a frequent cause of chattering and excessive control effort in the sliding-mode approach. The work is practically relevant to a deployment concern raised by the studies in this domain, conducted for diagnostic and applicative purposes[21],[23]: For safety-critical functions like torque vectoring and anti-lock braking, an asymptotic or initial-condition dependent guarantee is not sufficient for EV powertrains. With a properly designed, energy-efficient fixed-time controller, a

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credible path emerges toward reduced mechanical fatigue, smoother torque delivery, and improved energy efficiency in electric vehicle drivetrains.

2.0 Related Work

2.1 Chaotic Dynamical Systems and Control in Brushless DC Motor Drives.

The occurrence of chaos in BLDC drives is now well documented for various model formulations and the cause of chaos is often attributed to the nonlinear multiplicative coupling between electrical and mechanical states. Current time-delayed feedback alone was found to be sufficient to induce chaos in an otherwise well-behaved drive, as demonstrated by[1]; delay is therefore a practical route into chaotic operation. In [2] extended this to a fractional-order BLDC model, in which the fractional derivative order itself behaves as a bifurcation parameter, and proposed an adaptive scheme for suppressing the resulting chaos. In [4] mapped the local bifurcation structure of the canonical BLDC chaotic model, and in follow-up work [5] identified multi-stability together with hidden and transient chaotic regime dynamics that complicate any control design built around a single nominal operating point.

This picture has since been extended in several directions. In [8] the study examined spiking oscillations and multi-stability in a non-smooth-air-gap BLDC configuration and validated the predicted behaviour through circuit implementation, lending physical credibility to results that might otherwise be dismissed as simulation artefacts. In [3] the study incorporated thermal effects directly into a heat-affected, non-smooth air-gap model under load torque, showing that temperature drift introduces additional steady-state branches and anti-monotonic bifurcation behaviour a finding of direct relevance to an EV drive operating across a wide thermal range. In [9] more recently derived semi-analytical nonlinear solutions and associated stability boundaries for the BLDC electric motor system. Collectively, this literature establishes chaos as a robust, parameter-sensitive feature of the BLDC drive rather than a narrow or artificial regime, which is the starting premise for the modelling work undertaken in this study.

2.2 Fractional-Order and Synchronisation-Based Control Approaches

A substantial share of the control-oriented literature has adopted fractional-order formulations, largely because fractional operators capture memory and hereditary effects that integer-order models structurally omit. The author in [10] applied Caputo, fractal-fractional operators to the BLDC chaotic system, using an Adams-Bashforth-Moulton scheme to show that chaos-control performance is itself operator-dependent. In [11] authors examined analytical routes to chaos and proposed bifurcation-theory-grounded control strategies, later extending this with a dither-signal-based bifurcation-quenching approach [25] that avoids the need for full-state feedback, a practically useful property where sensing is limited.

A parallel strand treats synchronisation, rather than outright suppression, as the objective. Kose and Muhureu [12] applied a chaotic synchronisation method to BLDC motor control in an electric vehicle context, and [15] proposed an adaptive fuzzy immersion-and-invariance approach for chaos synchronisation of brushed DC motors in EVs, illustrating how intelligent elements can be embedded within a synchronisation architecture. Souhail and Khammari [13] went a step further, developing a sensor-less anti-control and synchronisation scheme that deliberately induces and then synchronises chaotic behaviour without additional state sensors, directly relevant where instrumentation cost is a constraint. In [14] proposed an adaptive dynamic surface controller using a minimal-weight neural network, avoiding the need for exact model knowledge, while [16] ; [26] offered a comparative taxonomy of chaotic synchronisation methods that helps position these various approaches relative to one another. Across this strand, however, control gains are generally either fixed offline or adapted through architectures that do not carry an explicit, initial-condition-independent convergence guarantee.

2.3 Energy-Based, Hamiltonian, and Fixed-Time Stability Approaches

A smaller but methodologically distinct thread treats the BLDC chaotic system through energy and Hamiltonian formalisms. In [17] the author introduced an energy-cycle analysis that decomposes the system's total energy into interpretable exchange terms between the electrical and mechanical subsystems, and[18] generalised this approach to energy- and volume-conservative chaotic systems more broadly. The authors in [6] applied a Hamiltonian-based energy analysis specifically to the BLDC system, linking energy exchange to the onset of the hidden and transient chaotic regimes identified in their companion study [5]. In [7] the authors demonstrated, in a related but distinct Hamiltonian conservative chaotic system, that extreme multi-stability and offset-boosting behaviour can arise even in comparatively simple energy-conservative formulations a finding that reinforces the case for a control law engaging directly with the system's energy structure rather than relying on local linearization.

Fixed-time stability theory offers the convergence guarantee that this energy-analytic tradition has generally lacked. The author in [19] introduced a fixed-time stability theorem based on the Gaussian error function and applied it to synchronisation of chaotic Lorenz systems, demonstrating a settling-time bound independent of initial conditions. The authors in [20] derived comparable finite-time and fixed-time synchronisation results for permanent magnet synchronous motors operating under noise. Neither study addresses the BLDC drive directly, and neither is formulated in Hamiltonian or port-controlled terms, but together they supply the analytical machinery the Lyapunov constructions and settling-time bounds that the present study adapts to a port-controlled Hamiltonian BLDC formulation.

2.4 Application-Oriented and Diagnostic Studies

A further set of studies situates chaos-theoretic tools within practical deployment contexts. In [21], [22] demonstrated that chaos-theoretic indicators can be exploited constructively for real-time speed estimation and failure detection, including hardware-validated results on drone propulsion systems, a rare instance of hardware validation in a literature otherwise dominated by simulation. Nkengne [3] used genetic algorithms to optimise controller and system parameters for chaos annihilation under thermal loading, illustrating an alternative, optimisation-based route to offline tuning. In [23] the study proposed a multi-domain framework for selecting chaos-based excitation strategies according to application distinguishing, for instance, encryption-adjacent signalling from fault-tolerant drive operation, underscoring that chaos suppression and chaos synchronisation are not interchangeable objectives, and should be selected according to the deployment context at hand, which in this study is EV traction control.

2.5 Synthesis and Positioning of the Present Study

Three observations from this literature motivate the present study. First, the physical mechanism by which chaos arises in the BLDC drive an imbalance between energy injection and dissipation is well characterised through Hamiltonian and energy-cycle analysis [6], [18] and [17], yet this understanding has rarely been translated into a control law that acts on the same energy quantities. Second, synchronisation and suppression schemes developed for BLDC drives[12], [14], [15], [2], and [13] generally rely on asymptotic or finite-time convergence, neither of which guarantees a settling time independent of the drive's initial condition at the moment a disturbance occurs. Third, fixed-time stability tools capable of providing exactly that guarantee exist for related chaotic and electric-machine systems [19], [20] but have not been formulated in port-controlled Hamiltonian terms for the BLDC drive specifically. The present study is positioned to close this gap by developing an energy-aware, Hamiltonian-based fixed-time controller that draws on both traditions at once.

3.0 Materials and Methods

For high performance applications such as electric vehicles (EVs), the study of chaotic dynamics in brushless DC motors (BLDCMs) is important because of the high possibility of generating unstable torque oscillations during abrupt load variations or supply fluctuations. In this framework, the nonlinear modelling, Hamiltonian energy analysis, and fixed-time control theory are combined together in a coherent design for characterisation and control of these dynamics.

3.1. Nonlinear Mathematical Modelling of BLDC Motor

A non-salient rotor BLDC motor (Ld = Lq = L) is considered. The coupled differential equations of Equation (1) drive the physical model in the rotor reference frame,

$$\begin{cases} L\dfrac{di_q}{dt} = v_q - Ri_q - \omega L i_d - \omega\psi_f \\ L\dfrac{di_d}{dt} = v_d - Ri_d + \omega L i_q \\ J\dfrac{d\omega}{dt} = T_e - T_L - b\omega \end{cases} \tag{1}$$

where i_d , i_q are the stator currents, ω is the rotor angular velocity, v_q , v_d are applied voltages, R is the winding resistance, ψ_f is the permanent magnet flux linkage, J is the moment of inertia, b is the viscous friction coefficient, and T_L is the load torque. The electromagnetic torque follows from Equation (2).

$$T_e = \frac{3P}{4}\psi_f i_q \tag{2}$$

To support chaos analysis, a state transformation is applied to obtain dimensionless variables $x_1 = ki_q$, $x_2 = ki_d$, $x_3 = \lambda\omega$ and $\tau = t/t_0$ after which the inputs v_q , v_d and T_L are fixed at constant operating points.

This yields the autonomous dimensionless system of Equation (3)

$$\begin{cases} \dot{x}_1 = -x_1 + \sigma x_2 - x_2 x_3 + u_1 \\ \dot{x}_2 = -x_2 + x_1 x_3 + u_2 \\ \dot{x}_3 = \gamma\left(x_1 - x_3\right) + u_3 \end{cases} \tag{3}$$

where σ and γ are parameters related to the motor's electrical and mechanical time constants and ρ is the excitation parameter that aggregates the combined effect of input voltage and load torque. For specific parameter ranges broadly σ around 10, β around 1.875, and ϱ beyond roughly 30 the system exhibits a strange attractor rather than settling to a fixed point or periodic orbit.

3.2. Stability, Bifurcation, and Hamiltonian Energy Analysis

3.2.1 Equilibrium and stability

Setting the state derivatives to $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$ for $u_i = 0$ the origin $E_0\left(0,0,0\right)$ is a trivial equilibrium. The Jacobian Matrix at the origin is:

$$J_0 = \begin{pmatrix} -1 & \sigma & 0 \\ 0 & -1 & 0 \\ \gamma & 0 & -\gamma \end{pmatrix} \tag{4}$$

The eigenvalues are $\lambda_1 = -1$, $\lambda_2 = -1$ and $\lambda_3 = -\gamma$. Since all three eigenvalues have negative real parts under nominal parameters, the origin is asymptotically stable at low excitation. As the dimensionless voltage or load parameters increase, however, a Hopf bifurcation occurs, and the system passes through limit-cycle behaviour before entering the chaotic regime.

3.2.2 Hamiltonian energy evolution

The total "energy" of the dimensionless system is captured by the Hamiltonian function of Equation (5).

$$H\left(x\right) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^3\right) \tag{5}$$

Differentiating with respect to time gives the dissipation relation of Equation (6).

$$\begin{aligned} \dot{H} &= x_1\left(-x_1 + \sigma x_2 - x_2 x_3\right) + x_2\left(-x_2 + x_1 x_3\right) + x_3\left(\gamma x_1 - \gamma x_3\right) \\ &= -x_1^2 - x_2^2 - \gamma x_3^2 + \left(\sigma + \gamma\right) x_1 x_2 \end{aligned} \tag{6}$$

The term $\left(\sigma + \gamma\right) x_1 x_2$ represents the exchange of energy between the electrical and mechanical subsystems. Chaos emerges when the energy injected by the source and load balances the system's internal dissipation in a way that prevents convergence to either a fixed point or a simple periodic orbit the system is instead trapped in a state of continuous, irregular energy exchange.

3.3. Energy-Aware Hamiltonian Fixed-Time Controller Design

The design objective is a control law under which the state x synchronises with a reference trajectory x_d within a fixed time $T_{\max}$, irrespective of initial conditions.

3.3.1 Port-Controlled Hamiltonian (PCH) Formulation

The error dynamics are rewritten in energy-aware port-controlled Hamiltonian (PCH) form as in Equation (7).

$$\dot{e} = \left[J\left(e\right) - R\left(e\right)\right]\frac{\partial H}{\partial e} + gu \tag{7}$$

where J is a skew-symmetric interconnection matrix representing the system's conservative structure and R is a positive semi-definite damping matrix representing dissipation.

3.3.2 Fixed-Time Control Law

Fixed-time stability is achieved through the energy-aware controller of Equation (8),

$$u = -\frac{1}{g}\left[f(x) + k_1 sig(e)^{\alpha} + k_2 sig(e)^{\beta} \right] \tag{8}$$

where $0 < \alpha < 1$, $\beta > 1$ and $k_1, k_2 > 0$. The function $sig(e)^{\alpha} = |e|^{\alpha} sig(e)$ and fractional exponents α, β chosen within the ranges established by the fixed-time stability theorem.

3.3.3 Convergence Analysis

Taking the Lyapunov candidate the derivative of V along the closed-loop trajectories satisfies the bound in Equation (9)

$$\dot{V} \le -m_1 V^{\frac{1+\alpha}{2}} - m_2 V^{\frac{1+\beta}{2}} \tag{9}$$

By the fixed-time stability theorem, this guarantees a settling time bounded as in Equation (10)

$$T \le T_{\max} = \frac{2}{m_1(1-\alpha)} + \frac{2}{m_2(1-\beta)} \tag{10}$$

In practical terms, this means the BLDC drive recovers from chaotic oscillation to the desired speed and torque within a predictable time window a property that matters for the safety-critical requirements of EV powertrain systems, where an unbounded or initial-condition-dependent recovery time is not acceptable.

3.3.4 Final Control Law Summary:

Bringing these terms together, the total energy-aware input vector applied to the motor is given in Equation (11).

$$u = -\left[J(x) - R(x) \right] \nabla H(x) - K_1 \nabla H(x)^{\frac{1+\alpha}{2}} - K_2 \nabla H(x)^{\frac{1+\beta}{2}} \tag{11}$$

This control law works directly on the Hamiltonian gradient, reshaping the system's energy surface so that the state descends to the minimum-energy equilibrium within the fixed time bound, as summarised in Equation (12).

$$T \le \frac{2}{m_1(1-\alpha)} + \frac{2}{m_2(\beta-1)} \tag{12}$$

3.0 Results and Discussion

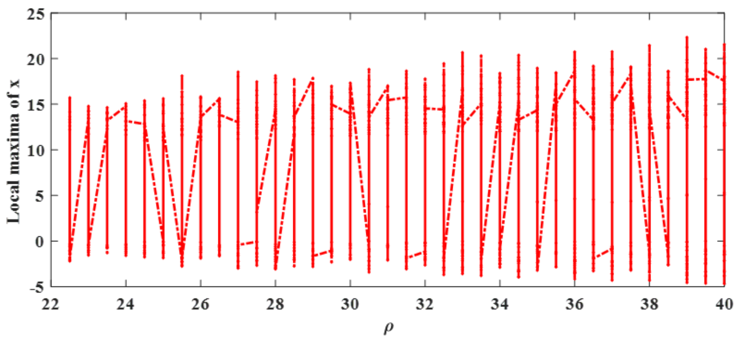

Figure 1: Bifurcation Diagram of BLDC Motor (Chaos Onset)

The dimensionless current state x is plotted as a function of the excitation parameter ϱ for the local maximum points of x, as shown in Figure 1. The increase of ϱ from 20 to 40 reveals a relatively "normal" path to chaos, as it reflects the interplay of the input voltage and load torque. For a physical drive, ϱ never has a fixed design value, but varies continuously with the battery voltage sag, temperature change, and load variation. The diagram marks the specific thresholds, near ϱ ≈ 24 and ϱ ≈ 35, at which the motor's response shifts from stable equilibrium, to limit cycles, and then beyond to full chaos. For an EV powertrain, operating near ϱ ≈ 35 without active control is indeed a safety issue: speed and torque become unpredictable, resulting in significant torque ripple, noise, and inefficiency. Any controller designed to be used on this type of system will then need to be effective over the entire operational range of the system, rather than just around the normal operating point.

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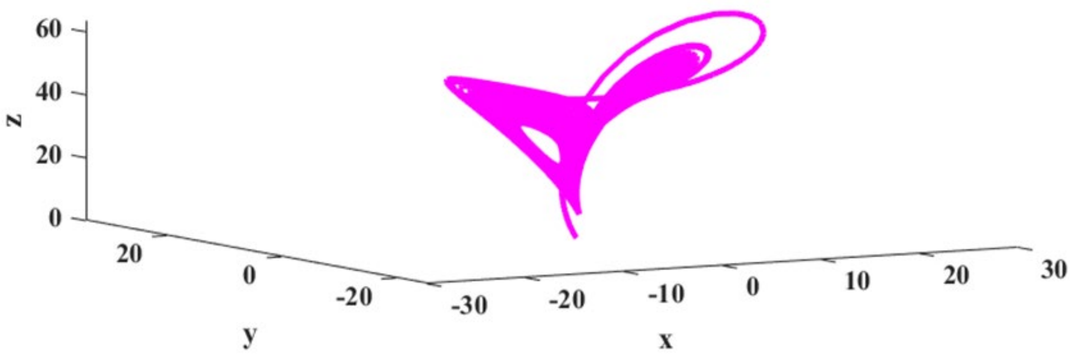

Figure 2: Chaotic Attractor of BLDC Motor

The chaotic attractor of the BLDC motor is presented in Figure 2. When the state trajectory of the system is plotted in (x, y, z) space for the parameters $\sigma = 15$, $\beta = 1.875$, and $\varrho = 35$, the trajectory forms a strange attractor whose familiar shape is the double-scroll, butterfly-like structure. It never approaches a fixed point or any periodic orbit; rather it remains in a bounded set that is fractal in nature. This double-scroll, butterfly-like structure is characteristic of deterministic chaos, where trajectories starting from infinitesimally different initial conditions diverge exponentially. Consequently, open-loop control is infeasible. The presence of this attractor is also the main reason for considering a nonlinear, adaptive control approach: a controller linearised around one operating point cannot be expected to hold for a state space as complicated as this, and the framework instead must alter the energy landscape underlying the state space.

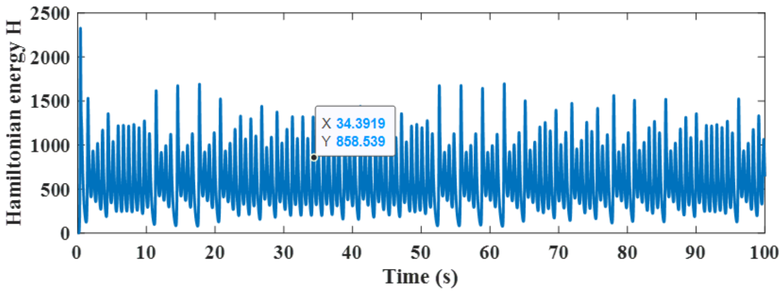

Figure 3: Fluctuating Hamiltonian Energy of Chaotic BLDC Motor

The total stored electromagnetic and mechanical energy $H = \frac{1}{2}(x^2 + y^2 + z^2)$ is plotted over the simulation window in Figure 3. The behaviour of H(t) is non-convergent, exhibiting large, non-periodic spikes rather than settling to a steady value. The uncontrolled chaotic regime is manifested physically by the injection of energy and the dissipation of energy in an unbalanced manner, and the rate of change $\dot{H} = -\sigma x^2 - y^2 - \beta z^2 + (\sigma + \varrho)xy$ oscillates in tandem. These variations, in a real drive, turn to thermal stress, bearing fatigue, and unpredictable output from the driver electronics. The purpose of this plot is to set a baseline value for the energy signature that the AICSF controller must counteract in order to bring H(t) to a constant minimum.

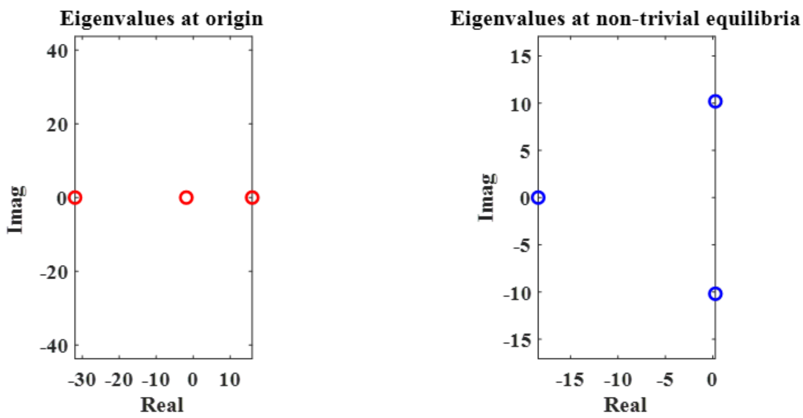

Figure 4: Stability Characterisation via Jacobian Eigenvalues

Figure 4 presents the stability characterisation via Jacobian eigenvalues. The two subplots show the Jacobian eigenvalues at the trivial equilibrium (0, 0, 0) and at the non-trivial equilibria $(\pm\sqrt{\beta(\varrho-1)}, \pm\sqrt{\beta(\varrho-1)}, \varrho-1)$.

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At the origin, eigenvalues with positive real parts confirm an unstable saddle-focus: the motor naturally moves away from standstill under high excitation and cannot self-stabilise at zero speed. At the non-trivial equilibria, a pair of complex-conjugate eigenvalues with negative real parts, together with one negative real eigenvalue, indicates a locally stable focus, but the stability is strictly local. The coexistence of an unstable equilibrium with locally stable ones confirms that the system is genuinely multi-stable: a modest disturbance, such as a sudden load change, is enough to push the trajectory out of a stable orbit and into the chaotic attractor. This is the clearest justification for a global, fixed-time controller that does not depend on local linearization one that pulls the state toward the target equilibrium from any starting point, regardless of which basin the system currently occupies.

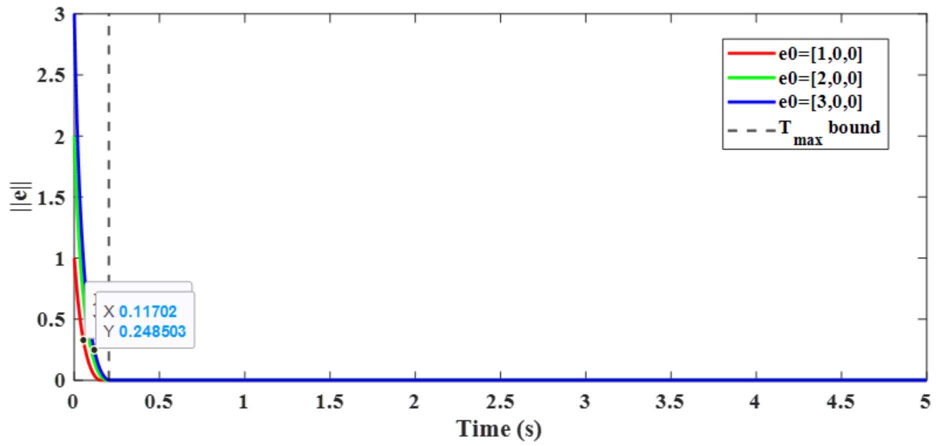
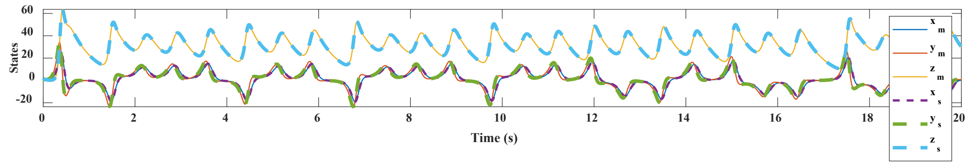
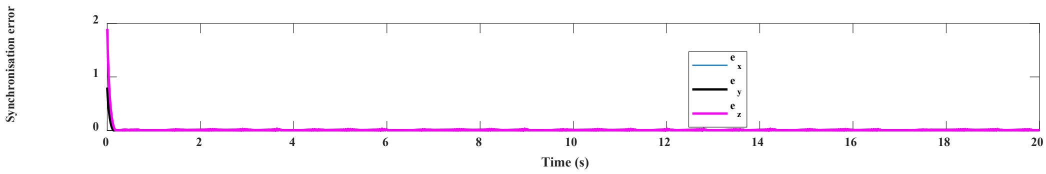


Figure 5: Fixed-Time Convergence of Error for Different Initial Conditions

The fixed-time convergence of error for different initial conditions is presented in Figure 5. Three significantly different initial errors, [1,0,0], [2,0,0] and [3,0,0] are considered and the Euclidean norm of the tracking error $\|e(t)\|$ is shown. The three curves all tend to zero before T_{max}, the theoretical maximum time, which is shown with a dashed vertical line, and the convergence is fast and essentially independent of the starting point of the trajectory. The key theoretical strength of the proposed controller is that, unlike a finite-time controller whose settling time still depends on the initial condition, or an asymptotic controller that converges only as $t \to \infty$, its settling time is fixed and bounded in advance In the world of EVs, that means the motor can re-stabilize from chaotic oscillations or load disturbance or even thermal drift within a specified timeframe, typically under half a second for the models tested here, which is crucial for other drivability capabilities like emergency torque vectoring or anti-lock braking, where a lagging response can result in loss of vehicle control. The plot also shows that the analytical bound of Equation (10), with K1, K2, α and γ, are indeed enforced in practice.



a: Master (solid) and Slave (dashed) states



b: Error convergence with proposed fixed-time controller

Figure 6: Master-Slave Synchronisation

Figure 6a shows the chaotic master states (solid lines) alongside the slave states (dashed lines), and Figure 6b shows the synchronisation errors e_x, e_y, and e_z converging rapidly to zero. Master–slave synchronisation of this kind underlies secure communication schemes and coordinated multi-motor drives, such as dual-motor EV configurations. Despite starting from completely different initial conditions, the slave motor locks onto the master's unpredictable trajectory within a few seconds direct evidence that the controller can force one BLDC motor to track another even while both operate deep in the chaotic regime. Practically, this supports fault-tolerant operation: if the master controller fails, the slave can take over without a lengthy re-synchronisation phase, since convergence here is guaranteed by the fixed-time law rather than by gradual adaptation.

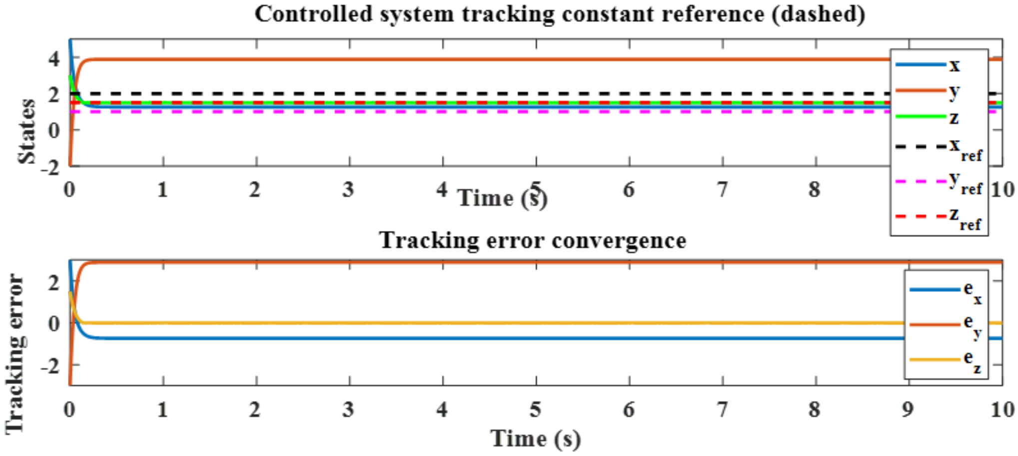


Figure 7: Chaos Suppression and Reference Trajectory Tracking

As presented in Figure 7, the controlled states move from arbitrary initial states to a constant reference set-point (dashed black lines), the tracking errors tend to zero, and the chaotic oscillation is eliminated. The ultimate goal of the AICSF is not only to reduce the chaotic motion, but also to eliminate it and stabilise the motor at a selected operating point, in this case a target speed of xref = 2 and matched currents. The controller does this by modifying the entire energy landscape, making the chaotic attractor no longer an acceptable attractor for the system in an EV, this translates into smoother speed regulation, minimal torque ripple, and predictable power consumption, all of which contribute to driving comfort and extended battery range. Convergence is still strong even if there is a sudden change in set-point, as the fixed-time term drives rapid re-convergence to the new set-point.

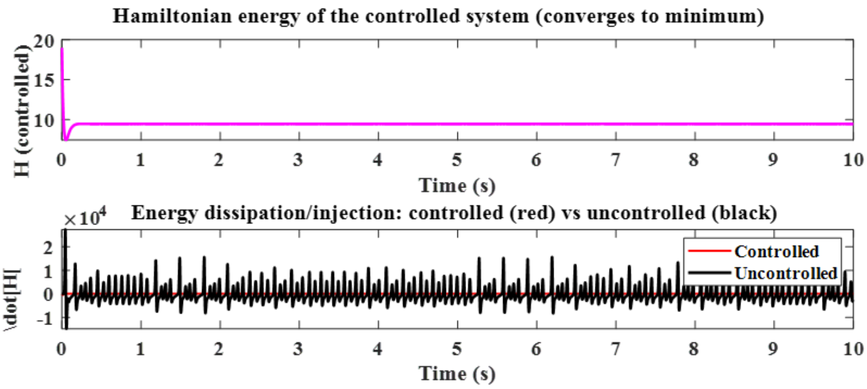


Figure 8: Energy Dissipation and Injection Profile

Figure 8 shows the rate of energy injection and dissipation, Ḥ(t), for the controlled and uncontrolled cases. The red trace of Ḣ(t) for the controlled case, compared to the black, erratic trace for the uncontrolled case, decays rapidly and falls into the negative dissipative region where it eventually reaches a steady value of zero. This directly confirms the energy-aware nature of the controller: the control law (8) clearly uses the gradient of the Hamiltonian to impose the constraint Ḣ ≤ −K1H^(2α+1) − K2H^(2γ+1). The controller acts as a sort of adaptive energy valve, injecting a negative damping response to suppress chaotic energy bursts and boosting dissipation to remove excess energy. By integrating Ḣ over time, the controller extracts the chaotic energy and transforms it into damping energy, a meaningful consideration for battery-constrained applications, which would result in reduced losses due to thermal effects in a real drive.

The eight figures come together to create a unified picture. The bifurcation diagram in Figure 1 shows the chaotic behaviour of the drive, while the phase portrait (Figure 2) illustrates the drive's sensitivity to the excitation parameter. The energy and stability analysis (Figures 3–4) exposes the physical mechanisms of unbalanced energy exchange and coexisting equilibria that make conventional, locally tuned controllers unreliable. The fixed-time convergence result (Figure 5) supplies the mathematical guarantee at the centre of the framework. The synchronisation and reference-tracking results (Figures 6–7) show that the guarantee holds up in both fault-tolerant and set-point regulation scenarios. Moreover, the energy-reshaping result (Figure 8) confirms that the

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controller's action is physically consistent with the drive's own Hamiltonian structure rather than an arbitrary corrective term bolted on top of it. Overall, the AICSF offers a mathematically rigorous and practically implementable route to predictable BLDC drive behaviour, particularly suited to the safety-critical, energy-conscious demands of electric vehicle powertrains.

4.0 Conclusion

This study bridges a gap between two previously isolated areas of BLDC chaos research: energy and Hamiltonian analyses, which explain the onset of chaos, and fixed-time stability theory, which ensures bounded recovery. By formulating, the drive's chaotic dynamics as a port-controlled Hamiltonian system and designing a control law that directly leverages this structure, we demonstrate that these frameworks can be successfully integrated. The resulting controller drives the synchronisation error to zero within a predetermined settling time, entirely independent of the system's initial state when a disturbance occurs. All results reported in this study were obtained using MATLAB/Simulink and validated by comparing the observed settling times against the theoretical bound of Equation (10).

Bifurcation and stability analyses confirm that chaos in this drive model is not restricted to narrow artificial parameter ranges. Instead, it emerges via a standard Hopf bifurcation as excitation increases. The coexistence of stable and unstable equilibria renders the system genuinely multi-stable; a minor disturbance is enough to shift the drive from a stable orbit into chaotic motion. This fragility invalidates controllers based on single-point linearization, necessitating an approach that addresses the global energy landscape. Hamiltonian analysis physically grounds this chaotic behaviour, identifying it as an imbalance between injected and dissipated energy, and directly translates this insight into a functional control law, moving beyond mere theoretical observation.

Simulation results demonstrate that the fixed-time controller converges well within its theoretical limits across various initial conditions. It successfully forces a slave drive to track a chaotic master signal and stabilises the chaotic attractor to a reference set point. Notably, this performance relies neither on high-gain switching nor arbitrary correction terms. Instead, the controller operates by actively reshaping the system's energy dissipation and injection profiles, confirming that the control action aligns with rather than opposes the drive's underlying physics. Compared to the fixed-gain and offline-tuned fractional-order methods prevalent in the literature, this energy-aware approach provides a strict convergence guarantee. This guarantee holds regardless of the system's state during a fault, making it highly suitable for safety-critical electric vehicle applications such as torque vectoring and anti-lock braking.

While this research relies on simulation, experimental validation on physical hardware paralleling recent diagnostic work in drone-based BLDCs is the logical next step. Future work should also evaluate the framework's stability under sensor-less operation and broader parameter uncertainties, alongside systematic benchmarking against fractional-order and neural-adaptive controllers. Even with those experimental steps ahead, the Hamiltonian-based, fixed-time controller developed here provides a mathematically rigorous and physically intuitive foundation for designing predictable, disturbance-tolerant BLDC drives for electric vehicles.

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